Loans & investments with regular payments

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** List the numbers in the words: the loan or savings amount, the payment, the yearly rate, how often interest is compounded (C/Y), how often you pay (P/Y), and how many years.
2. 🎯 **Asked** Which one is blank: the payment PMT, the loan size PV, or the savings total FV? Does the question also want the total paid or the interest?
3. ✏️ **Picture it** Draw a timeline: one arrow for each payment at the end of each period. Money today sits at the left (PV); money at the end sits at the right (FV).
4. 🧰 **Tool** i = rate per PAYMENT period, n = number of payments. Saving: FV formula. Loan size: PV formula. Loan payment: PMT formula. Or fill the TVM list, money out negative.
5. ✅ **Check** Payments times n must be MORE than the loan (you pay interest) and LESS than the savings total (interest is added). The difference is the interest.

⭐ Must know

- Saving with equal deposits → FV. Loan → amount borrowed is PV; solve for PMT.
- i = rate per PAYMENT period, n = number of payments (monthly, 5 years → $n = 60$).
- Interest = total paid − amount borrowed (or − total deposited).

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i}$$

$$PV = PMT \cdot \frac{1 - (1+i)^{-n}}{i}$$

🔍 Watch out for

1. Using the yearly rate as the monthly rate
2. Counting years instead of payments
3. Calling the total paid the interest
4. Using r over 12 for a Canadian mortgage
5. Using the savings formula to find a loan

🌱 Small wins — tick as you go

- ○ I can turn a yearly percent into a monthly decimal rate and type a power with a negative exponent into my calculator. (6′)
- ○ I can list PV or FV, PMT, i and n from the words and say which one is missing. (6′)
- ○ I can follow, then finish, a loan solution: payment, total paid, interest. (10′)
- ○ I can find a loan payment and its interest on my own with at most two hints. (8′)
- ○ I can spot the five classic annuity mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions and say whether each answer is reasonable. (7′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. An account pays 4.8% per year, compounded monthly. What is the monthly rate as a decimal?
A. 0.048 B. 0.4 C. 0.004 D. 0.0004

If you missed it — **Percent, decimals and rates**: Percent to decimal: divide by 100 ($4.5\% = 0.045$). A rate per year split into n periods is $\frac{r}{n}$ per period: 6% compounded monthly is 0.005 per month. Increase by r: multiply by $(1 + r)$.

1. $\$200$ increased by 3%: $200 \times 1.03 = 206$.
2. 3.6% compounded quarterly: $\frac{0.036}{4} = 0.009$ per quarter.

2. $\$5000$ earns 4% per year, compounded once a year, for 2 years. How much is in the account at the end?
A. $\$5400$ B. $\$5408$ C. $\$5800$ D. $\$5200$

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1. $\$200$ increased by 3%: $200 \times 1.03 = 206$.
2. 3.6% compounded quarterly: $\frac{0.036}{4} = 0.009$ per quarter.

3. Evaluate 1.02^{-2}, to 4 decimal places.
A. 0.9612 B. -1.0404 C. 1.0404 D. 0.9600

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3 x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

4. Evaluate $\dfrac{1 - 1.01^{-12}}{0.01}$, to 2 decimal places.
A. 11.26 B. 12.68 C. 0.11 D. -11.26

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1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

Answers: 1: C · 2: B · 3: A · 4: A

🔍 Common-errors sheet

1. Using the yearly rate as the monthly rate

A $\$6000$ loan at 6% per year compounded monthly is repaid monthly over 2 years. Find the payment.

1 $i = 0.06,\ n = 24$

2 $PMT = \dfrac{6000(0.06)}{1 - 1.06^{-24}} \approx \$478.07$

✍ Which line goes wrong first, and why?

2. Counting years instead of payments

You save $\$50$ at the end of every month for 6 years at 3% compounded monthly. Find the future value.

1 $i = \dfrac{0.03}{12} = 0.0025,\ n = 6$

2 $FV = 50 \cdot \dfrac{1.0025^{6} - 1}{0.0025} \approx \$301.88$

✍ Which line goes wrong first, and why?

3. Calling the total paid the interest

A $\$10\,000$ loan at 5% per year compounded monthly is repaid with monthly payments of $\$299.71$ for 3 years. How much interest is paid?

1 Total of the payments: $36 \times 299.71 = \$10\,789.56$

2 Interest $= \$10\,789.56$

✍ Which line goes wrong first, and why?

4. Using r over 12 for a Canadian mortgage

A $\$250\,000$ mortgage at 6% per year compounded semi-annually is amortized over 25 years with monthly payments. Find the payment.

1 $i = \dfrac{0.06}{12} = 0.005,\ n = 300$

2 $PMT = \dfrac{250\,000(0.005)}{1 - 1.005^{-300}} \approx \$1610.75$

✍ Which line goes wrong first, and why?

5. Using the savings formula to find a loan

You can afford $\$400$ a month for 5 years. At 6% compounded monthly, what is the largest loan you can take?

1 Use $FV = 400 \cdot \dfrac{1.005^{60} - 1}{0.005}$

2 $\approx \$27\,908.01$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1** — The question shows "6%", so 0.06 goes straight in. But interest is added every MONTH, so each month gets only one twelfth of it. $i = 0.005$, $n = 24$: $PMT = \dfrac{6000(0.005)}{1 - 1.005^{-24}} \approx \$265.92$. (Sense check: $6000 \div 24 = 250$, so 478 is far too high.)
2. **line 1** — "6 years" is the number in the question, so it becomes n. But n counts deposits, and there is one every month. $n = 12 \times 6 = 72$: $FV = 50 \cdot \dfrac{1.0025^{72} - 1}{0.0025} \approx \$3938.97$. (You deposit $\$3600$ yourself, so $\$301.88$ cannot be right.)
3. **line 2** — All that money leaves your account, so it feels like the cost. But most of it is paying back the $\$10\,000$ you borrowed; only the extra is interest. Interest $= 10\,789.56 - 10\,000 = \$789.56$.
4. **line 1** — "Paid monthly, so divide by 12" works only when interest is ALSO added monthly. Here interest is added twice a year, so the monthly rate must grow to 3% over 6 months. $i = 1.03^{1/6} - 1 \approx 0.0049386$, so $PMT \approx \$1599.52$. On a TVM solver: P/Y $= 12$, C/Y $= 2$.
5. **line 1** — Both formulas look alike, and FV is the one used most. But a loan is money you get TODAY, at the left end of the timeline, so it is a present value. $PV = 400 \cdot \dfrac{1 - 1.005^{-60}}{0.005} \approx \$20\,690.22$. (Sense check: you pay $400 \times 60 = \$24\,000$ in total, so the loan must be LESS than that.)