Compound interest

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** List the numbers: principal P, yearly rate r as a DECIMAL, times per year it compounds n, and years t. Read the words for n: annually 1, semi-annually 2, quarterly 4, monthly 12.
2. 🎯 **Asked** Is the unknown the final amount A, the interest $A - P$, or the amount to invest now, P? Money is rounded to the cent, at the very end.
3. ✏️ **Picture it** A growth table: each period the balance is multiplied by $(1 + i)$. After N periods it has been multiplied N times.
4. 🧰 **Tool** $i = \frac{r}{n}$ and $N = nt$, then $A = P(1 + i)^N$ typed in ONE line with brackets. To find P, divide A by $(1 + i)^N$. ("Compounded semi-annually", the usual way a Canadian fixed-rate mortgage is quoted, means $n = 2$.)
5. ✅ **Check** A must be bigger than P, and a little bigger than simple interest $P(1 + rt)$. The Rule of 72 gives a rough size.

⭐ Must know

- $A = P\left(1 + \frac{r}{n}\right)^{nt}$: r as a decimal, n = periods per year, exponent nt.
- Per period $i = \frac{r}{n}$; periods $= nt$ (monthly for 6 years = 72).
- Rule of 72: doubling time $\approx \frac{72}{\text{rate in \%}}$ years — an estimate only.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

🔍 Watch out for

1. Using the percent as a whole number
2. Using years instead of periods as the exponent
3. Forgetting to divide the rate by n
4. Working out simple interest instead
5. Rounding the rate per period too early

🌱 Small wins — tick as you go

- ○ I can write a percent as a decimal and split a yearly rate into a rate per period. (5′)
- ○ I can list P, r, n and t from a word problem and say whether A, P or the interest is wanted. (5′)
- ○ I can follow, then finish, a solution for the final amount and the interest. (8′)
- ○ I can find a final amount on my own with at most two hints, typing it in one line. (7′)
- ○ I can spot the five classic compound-interest mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions and say whether each answer is reasonable. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Write 4.5% as a decimal.
A. 4.5 B. 0.45 C. 0.045 D. 0.0045

If you missed it — **Percent, decimals and rates**: Percent to decimal: divide by 100 ($4.5\% = 0.045$). A rate per year split into n periods is $\frac{r}{n}$ per period: 6% compounded monthly is 0.005 per month. Increase by r: multiply by $(1 + r)$.

1. $\$200$ increased by 3%: $200 \times 1.03 = 206$.
2. 3.6% compounded quarterly: $\frac{0.036}{4} = 0.009$ per quarter.

2. A rate is 6% per year, compounded quarterly. What is the rate per quarter, as a decimal?
A. 0.06 B. 0.015 C. 0.24 D. 0.15

If you missed it — **Percent, decimals and rates**: Percent to decimal: divide by 100 ($4.5\% = 0.045$). A rate per year split into n periods is $\frac{r}{n}$ per period: 6% compounded monthly is 0.005 per month. Increase by r: multiply by $(1 + r)$.

1. $\$200$ increased by 3%: $200 \times 1.03 = 206$.
2. 3.6% compounded quarterly: $\frac{0.036}{4} = 0.009$ per quarter.

3. Evaluate 1.05^3 to 4 decimal places.
A. 1.1576 B. 1.15 C. 3.15 D. 1.1025

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

4. Which number equals $2000(1.01)^2$?
A. 2040.20 B. 2040 C. $4\,080\,400$ D. 2020

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

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Answers: 1: C · 2: B · 3: A · 4: A

🔍 Common-errors sheet

1. Using the percent as a whole number

$\$1000$ is invested at 4% compounded annually for 3 years. Find the amount.

1 $r = 4$

2 $A = 1000(1 + 4)^3 = 125\,000$

✍ Which line goes wrong first, and why?

2. Using years instead of periods as the exponent

$\$2000$ is invested at 6% compounded monthly for 4 years. Find the amount.

1 $i = 0.06 \div 12 = 0.005$

2 $A = 2000(1.005)^4 \approx 2040.30$

✍ Which line goes wrong first, and why?

3. Forgetting to divide the rate by n

$\$1500$ is invested at 8% compounded quarterly for 3 years. Find the amount.

1 $i = 0.08$, $N = 4 \times 3 = 12$

2 $A = 1500(1.08)^{12} \approx 3777.26$

✍ Which line goes wrong first, and why?

4. Working out simple interest instead

$\$5000$ is invested at 3% compounded annually for 4 years. Find the amount.

1 Interest $= 5000(0.03)(4) = 600$

2 $A = 5000 + 600 = 5600$

✍ Which line goes wrong first, and why?

5. Rounding the rate per period too early

$\$6000$ is invested at 5% compounded monthly for 10 years. Find the amount.

1 $i = 0.05 \div 12 \approx 0.004$

2 $A = 6000(1.004)^{120} \approx 9687.17$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1** — 4% means 4 out of 100, so $r = 0.04$. With $r = 4$ the money would grow 400% a year. $A = 1000(1.04)^3 \approx \$1124.86$.
2. **line 2** — The balance is multiplied by 1.005 once a MONTH. Four years is $12 \times 4 = 48$ months, so the exponent is 48. $A = 2000(1.005)^{48} \approx \$2540.98$.
3. **line 1** — 8% is the rate for a whole YEAR. Each quarter earns only a quarter of it: $i = 0.08 \div 4 = 0.02$. $A = 1500(1.02)^{12} \approx \$1902.36$. (The wrong answer, $\$3777.26$, more than doubles the money in 3 years, which should look suspicious.)
4. **line 1** — That gives interest on the first $\$5000$ only. With compounding, each year's interest also earns interest. $A = 5000(1.03)^4 \approx \$5627.54$, about $\$27.54$ more than simple interest.
5. **line 1** — $0.05 \div 12 = 0.0041666\ldots$ Cutting it to 0.004 looks tiny, but the error is raised to the power 120. Type it in one line: $6000(1 + 0.05/12)^{120} \approx \$9882.06$, about $\$195$ more.