Compound interest

复利

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** List the numbers: principal P, yearly rate r as a DECIMAL, times per year it compounds n, and years t. Read the words for n: annually 1, semi-annually 2, quarterly 4, monthly 12.

   > 列出数字：本金 P、年利率 r（写成「小数」）、每年计息次数 n、年数 t。从文字里读出 n：按年 1，半年 2，按季 4，按月 12。

2. 🎯 **Asked · 求什么** Is the unknown the final amount A, the interest $A - P$, or the amount to invest now, P? Money is rounded to the cent, at the very end.

   > 要求的是终值 A、利息 $A - P$，还是现在要投入的 P？金额四舍五入到分，而且只在最后一步。

3. ✏️ **Picture it · 画出来** A growth table: each period the balance is multiplied by $(1 + i)$. After N periods it has been multiplied N times.

   > 一张增长表：每过一期，余额乘一次 $(1 + i)$。过了 N 期就乘了 N 次。

4. 🧰 **Tool · 用什么工具** $i = \dfrac{r}{n}$ and $N = nt$, then $A = P(1 + i)^N$ typed in ONE line with brackets. To find P, divide A by $(1 + i)^N$. ("Compounded semi-annually", the usual way a Canadian fixed-rate mortgage is quoted, means $n = 2$.)

   > $i = \dfrac{r}{n}$，$N = nt$，再把 $A = P(1 + i)^N$ 带括号「一行」输入计算器。求 P 就用 A 除以 $(1 + i)^N$。（「半年复利」——加拿大固定利率房贷通常这样标——就是 $n = 2$。）

5. ✅ **Check · 检验** A must be bigger than P, and a little bigger than simple interest $P(1 + rt)$. The Rule of 72 gives a rough size.

   > A 一定比 P 大，而且比单利 $P(1 + rt)$ 稍大一点。72 法则可以估个大概。

⭐ Must know

必须记住

- $A = P\left(1 + \frac{r}{n}\right)^{nt}$: r as a decimal, n = periods per year, exponent nt.
  > $A = P\left(1 + \frac{r}{n}\right)^{nt}$：$r$ 用小数，n 是每年计息次数，指数是 nt。
- Per period $i = \frac{r}{n}$; periods $= nt$ (monthly for 6 years = 72).
  > 每期利率 $i = \frac{r}{n}$；期数 $= nt$（按月 6 年 $= 72$ 期）。
- Rule of 72: doubling time $\approx \frac{72}{\text{rate in \%}}$ years — an estimate only.
  > 72 法则：翻倍时间 $\approx \frac{72}{\text{年利率（\%）}}$ 年——只是估算。

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

🔍 Watch out for

当心

1. Using the percent as a whole number
   > 把百分数当整数用
2. Using years instead of periods as the exponent
   > 指数用了年数而不是期数
3. Forgetting to divide the rate by n
   > 忘了把利率除以 n
4. Working out simple interest instead
   > 算成了单利
5. Rounding the rate per period too early
   > 每期利率舍入太早

🌱 Small wins — tick as you go

一小步一小步——做完打勾

○ I can write a percent as a decimal and split a yearly rate into a rate per period.
> 我会把百分数写成小数，也会把年利率分成每期利率。

(5′)

○ I can list P, r, n and t from a word problem and say whether A, P or the interest is wanted.
> 我能从题目文字里列出 P、r、n、t，并说出要求的是 A、P 还是利息。

(5′)

○ I can follow, then finish, a solution for the final amount and the interest.
> 我能看懂并补全一道求终值和利息的题。

(8′)

○ I can find a final amount on my own with at most two hints, typing it in one line.
> 我能在最多两个提示下独立求出终值，并在计算器上一行输完。

(7′)

○ I can spot the five classic compound-interest mistakes in someone else's work.
> 我能在别人的解答里找出复利题的五个经典错误。

(8′)

○ I can do the Level 1–2 practice questions and say whether each answer is reasonable.
> 我能做出 1–2 级练习题，并判断每个答案是否合理。

(10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Write 4.5% as a decimal.

> 把 4.5% 写成小数。

A. 4.5 B. 0.45 C. 0.045 D. 0.0045

If you missed it · 如果做错了 — **Percent, decimals and rates**

> **百分数、小数与比率**
> : Percent to decimal: divide by 100 ($4.5\% = 0.045$). A rate per year split into n periods is $\frac{r}{n}$ per period: 6% compounded monthly is 0.005 per month. Increase by r: multiply by $(1 + r)$.
> 百分数化小数：除以 100（$4.5\% = 0.045$）。年利率分成 n 期，每期是 $\frac{r}{n}$：年利率 6% 按月复利，每月 0.005。增长 r：乘以 $(1 + r)$。

1. $\$200$ increased by 3%: $200 \times 1.03 = 206$.
   > $\$200$ 增长 3%：$200 \times 1.03 = 206$。
2. 3.6% compounded quarterly: $\frac{0.036}{4} = 0.009$ per quarter.
   > 年利率 3.6% 按季复利：每季 $\frac{0.036}{4} = 0.009$。

2. A rate is 6% per year, compounded quarterly. What is the rate per quarter, as a decimal?

> 年利率 6%，按季度复利。每季度的利率写成小数是多少?

A. 0.06 B. 0.015 C. 0.24 D. 0.15

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   > 年利率 3.6% 按季复利：每季 $\frac{0.036}{4} = 0.009$。

3. Evaluate 1.05^3 to 4 decimal places.

> 计算 1.05^3，保留 4 位小数。

A. 1.1576 B. 1.15 C. 3.15 D. 1.1025

If you missed it · 如果做错了 — **Exponent rules (product, quotient, power)**

> **指数运算法则（同底乘、除、乘方）**
> : Same base: multiply → ADD exponents ($x^3 x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.
> 同底数：相乘→指数相加（$x^3 x^4 = x^7$）；相除→指数相减（$x^7 \div x^2 = x^5$）；幂的乘方→指数相乘（$(x^3)^2 = x^6$）。积的乘方每个因数都要乘方：$(2x)^3 = 8x^3$。

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
   > $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$。
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).
   > $\frac{6x^5}{2x^2} = 3x^3$（系数相除，指数相减）。

4. Which number equals $2000(1.01)^2$?

> 哪个数等于 $2000(1.01)^2$?

A. 2040.20 B. 2040 C. $4\,080\,400$ D. 2020

If you missed it · 如果做错了 — **Exponent rules (product, quotient, power)**

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Answers · 答案: 1: C · 2: B · 3: A · 4: A

🔍 Common-errors sheet

常见错误单

1. Using the percent as a whole number

> **把百分数当整数用**

$\$1000$ is invested at 4% compounded annually for 3 years. Find the amount.

> $\$1000$ 以 4%、按年复利投资 3 年。求终值。

1 $r = 4$

2 $A = 1000(1+4)^3 = 125\,000$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Using years instead of periods as the exponent

> **指数用了年数而不是期数**

$\$2000$ is invested at 6% compounded monthly for 4 years. Find the amount.

> $\$2000$ 以 6%、按月复利投资 4 年。求终值。

1 $i = 0.06 \div 12 = 0.005$

2 $A = 2000(1.005)^4 \approx 2040.30$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Forgetting to divide the rate by n

> **忘了把利率除以 n**

$\$1500$ is invested at 8% compounded quarterly for 3 years. Find the amount.

> $\$1500$ 以 8%、按季度复利投资 3 年。求终值。

1 $i = 0.08,\ N = 4 \times 3 = 12$

> $i = 0.08,\ \ N = 4 \times 3 = 12$

2 $A = 1500(1.08)^{12} \approx 3777.26$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Working out simple interest instead

> **算成了单利**

$\$5000$ is invested at 3% compounded annually for 4 years. Find the amount.

> $\$5000$ 以 3%、按年复利投资 4 年。求终值。

1 Interest $= 5000(0.03)(4) = 600$

> 利息 $= 5000(0.03)(4) = 600$

2 $A = 5000 + 600 = 5600$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Rounding the rate per period too early

> **每期利率舍入太早**

$\$6000$ is invested at 5% compounded monthly for 10 years. Find the amount.

> $\$6000$ 以 5%、按月复利投资 10 年。求终值。

1 $i = 0.05 \div 12 \approx 0.004$

2 $A = 6000(1.004)^{120} \approx 9687.17$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 1 —** 4% means 4 out of 100, so $r = 0.04$. With $r = 4$ the money would grow 400% a year.

   > 4% 是百分之四，$r = 0.04$。用 $r = 4$ 等于每年涨 400%。

   $A = 1000(1.04)^3 \approx \$1124.86$.

   > $A = 1000(1.04)^3 \approx \$1124.86$。

2. **line · 第 2 —** The balance is multiplied by 1.005 once a MONTH. Four years is $12 \times 4 = 48$ months, so the exponent is 48.

   > 余额每「月」乘一次 1.005。4 年是 $12 \times 4 = 48$ 个月，指数应是 48。

   $A = 2000(1.005)^{48} \approx \$2540.98$.

   > $A = 2000(1.005)^{48} \approx \$2540.98$。

3. **line · 第 1 —** 8% is the rate for a whole YEAR. Each quarter earns only a quarter of it: $i = 0.08 \div 4 = 0.02$.

   > 8% 是「一整年」的利率。每个季度只拿四分之一：$i = 0.08 \div 4 = 0.02$。

   $A = 1500(1.02)^{12} \approx \$1902.36$. (The wrong answer, $\$3777.26$, more than doubles the money in 3 years, which should look suspicious.)

   > $A = 1500(1.02)^{12} \approx \$1902.36$。（错误答案 3777.26 加元，3 年就让钱翻了一倍多，这本身就该起疑。）

4. **line · 第 1 —** That gives interest on the first $\$5000$ only. With compounding, each year's interest also earns interest.

   > 这样只算了最初 $\$5000$ 的利息。复利时，每年的利息也会再生利息。

   $A = 5000(1.03)^4 \approx \$5627.54$, about $\$27.54$ more than simple interest.

   > $A = 5000(1.03)^4 \approx \$5627.54$，比单利多约 $\$27.54$。

5. **line · 第 1 —** $0.05 \div 12 = 0.0041666\ldots$ Cutting it to 0.004 looks tiny, but the error is raised to the power 120.

   > $0.05 \div 12 = 0.0041666\ldots$，舍成 0.004 看着差得不多，可这个误差被乘方了 120 次。

   Type it in one line: $6000(1 + 0.05/12)^{120} \approx \$9882.06$, about $\$195$ more.

   > 一行输入：$6000(1 + 0.05/12)^{120} \approx \$9882.06$，多出约 $\$195$。