Buy, lease or rent?

买、租赁还是租用?

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** For each option, list every cash flow over the SAME number of years: down payment, each payment times the number of months, end fees, and what you own at the end (its resale value).

   > 对每种方案，列出「相同年数」内的每一笔钱：首付、每期付款乘以月数、到期费用，以及期末拥有的东西（转卖价值）。

2. 🎯 **Asked · 求什么** Which option has the lower NET cost, and by how much? Then give a one-sentence recommendation with a reason.

   > 哪种方案的「净成本」更低？低多少？最后用一句话给出建议并说明理由。

3. ✏️ **Picture it · 画出来** A two-column table, Buy | Lease (or Rent | Own). One line per cost; the buy column ends with "− resale value".

   > 一张两栏表：买 | 租（或租房 | 买房）。每项成本一行；「买」那一栏最后一行是「− 转卖价值」。

4. 🧰 **Tool · 用什么工具** Resale value $V = P(1-r)^t$; appreciation $V = P(1+r)^t$. A loan payment comes from the payment formula or TVM. For a home, count only interest, tax, strata, maintenance and lost return; a Canadian fixed-rate mortgage is quoted compounded semi-annually (C/Y $= 2$, P/Y $= 12$).

   > 转卖价值 $V = P(1-r)^t$；升值 $V = P(1+r)^t$。贷款月供用月供公式或 TVM。房子只算利息、地税、物业费、维修和机会成本；加拿大固定利率房贷按半年复利标示（C/Y $= 2$，P/Y $= 12$）。

5. ✅ **Check · 检验** Same time span on both sides? Resale less than the price? A lower monthly payment does NOT mean a lower total.

   > 两边年数一样吗？转卖价低于原价吗？月供低「不等于」总成本低。

⭐ Must know

必须记住

- Buy: net cost = total paid − value at the end. Lease: down + payments + end fees; you own nothing.
  > 买：净成本＝总支出 − 期末残值。租：首付＋月供＋到期费用；到期什么都不归你。
- Depreciation compounds: $V = P(1 - r)^t$; appreciation: $V = P(1 + r)^t$.
  > 贬值是复利式的：$V = P(1 - r)^t$；升值：$V = P(1 + r)^t$。
- Rent vs buy: compare unrecoverable costs; mortgage principal is equity, not a cost.
  > 租房还是买房：比较收不回的成本；房贷本金是自己的资产，不算成本。

$$V = P(1 - r)^t$$

$$V = P(1 + r)^t$$

🔍 Watch out for

当心

1. Comparing only the monthly payments
   > 只比较月供
2. Forgetting to subtract the resale value
   > 忘了减去转卖价值
3. Taking the same amount off every year
   > 每年都减同样的钱
4. Multiplying by the rate instead of what is kept
   > 乘的是贬值率而不是保留率
5. Counting the whole mortgage payment as a cost
   > 把整笔房贷还款都算成本

🌱 Small wins — tick as you go

一小步一小步——做完打勾

○ I can turn "loses 15% a year" into the multiplier 0.85 and do a growth or a net-cost calculation.
> 我能把「每年贬值 15%」变成乘数 0.85，也会算增长和净成本。

(5′)

○ I can list every cost of both options over the same number of years, including what I own at the end.
> 我能列出两种方案在相同年数里的每一项花费，包括期末还拥有什么。

(5′)

○ I can follow, then finish, a buy-versus-lease comparison in a two-column table.
> 我能看懂并补全一张「买还是租」的两栏比较表。

(10′)

○ I can compare buying and leasing a car on my own with at most two hints.
> 我能在最多两个提示下，独立比较买车和租车。

(8′)

○ I can spot the five classic buy, lease and rent mistakes in someone else's work.
> 我能在别人的解答里找出买、租车与租房题的五个经典错误。

(8′)

○ I can do the Level 1–2 practice questions and end with a one-sentence recommendation.
> 我能做出 1–2 级练习题，并用一句话给出建议。

(10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. A car loses 15% of its value each year. Each year its value is multiplied by…

> 一辆车每年贬值 15%。它的价值每年乘以……

A. 0.15 B. 0.85 C. 1.15 D. 85

If you missed it · 如果做错了 — **Percent, decimals and rates**

> **百分数、小数与比率**

: Percent to decimal: divide by 100 ($4.5\% = 0.045$). A rate per year split into n periods is $\frac{r}{n}$ per period: 6% compounded monthly is 0.005 per month. Increase by r: multiply by $(1 + r)$.

> 百分数化小数：除以 100（$4.5\% = 0.045$）。年利率分成 n 期，每期是 $\frac{r}{n}$：年利率 6% 按月复利，每月 0.005。增长 r：乘以 $(1 + r)$。

1. $\$200$ increased by 3%: $200 \times 1.03 = 206$.
   > $\$200$ 增长 3%：$200 \times 1.03 = 206$。
2. 3.6% compounded quarterly: $\frac{0.036}{4} = 0.009$ per quarter.
   > 年利率 3.6% 按季复利：每季 $\frac{0.036}{4} = 0.009$。

2. Evaluate $20\,000(0.9)^2$.

> 计算 $20\,000(0.9)^2$。

A. $16\,200$ B. $16\,000$ C. $18\,000$ D. $324\,000\,000$

If you missed it · 如果做错了 — **Exponent rules (product, quotient, power)**

> **指数运算法则（同底乘、除、乘方）**

: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

> 同底数：相乘→指数相加（$x^3x^4 = x^7$）；相除→指数相减（$x^7 \div x^2 = x^5$）；幂的乘方→指数相乘（$(x^3)^2 = x^6$）。积的乘方每个因数都要乘方：$(2x)^3 = 8x^3$。

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
   > $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$。
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).
   > $\frac{6x^5}{2x^2} = 3x^3$（系数相除，指数相减）。

3. A home worth $\$400\,000$ rises 3% in a year. What is it worth then?

> 一套价值 $\$400\,000$ 的房子一年涨了 3%。现在值多少?

A. $\$412\,000$ B. $\$403\,000$ C. $\$1\,600\,000$ D. $\$388\,000$

If you missed it · 如果做错了 — **Percent, decimals and rates**

> **百分数、小数与比率**

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> 百分数化小数：除以 100（$4.5\% = 0.045$）。年利率分成 n 期，每期是 $\frac{r}{n}$：年利率 6% 按月复利，每月 0.005。增长 r：乘以 $(1 + r)$。

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   > $\$200$ 增长 3%：$200 \times 1.03 = 206$。
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   > 年利率 3.6% 按季复利：每季 $\frac{0.036}{4} = 0.009$。

4. You paid $\$21\,500$ in total for a car and sold it for $\$9800$. What was your net cost?

> 你买车一共付了 $\$21\,500$，后来卖了 $\$9800$。净成本是多少?

A. $\$11\,700$ B. $\$31\,300$ C. $-\$11\,700$ D. $\$12\,300$

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5-(-3)=5+3=8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2=16$, but $-4^2=-16$ (the square happens first).

> 减去负数等于加上它：$5-(-3)=5+3=8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2=16$，但 $-4^2=-16$（先算平方）。

1. $-7-(-2)$: change "minus negative" to "plus": $-7+2=-5$.
   > $-7-(-2)$：把「减负」改成「加」：$-7+2=-5$。
2. $(-3)(-5)=15$ (same signs), $(-3)(5)=-15$ (different signs).
   > $(-3)(-5)=15$（同号），$(-3)(5)=-15$（异号）。
3. Plugging in $x=-3$ to x^2: write brackets first, $(-3)^2=9$.
   > 把 $x=-3$ 代入 x^2：先加括号，$(-3)^2=9$。

Answers · 答案: 1: B · 2: A · 3: A · 4: A

🔍 Common-errors sheet

常见错误单

1. Comparing only the monthly payments

> **只比较月供**

A $\$26\,000$ car. Lease: $\$2000$ down, $\$329$ a month for 48 months, $\$500$ in fees. Buy: no down payment and a loan of $\$580$ a month for 48 months, then sell the car (it loses 15% a year). Which costs less over 4 years?

> 一辆 $\$26\,000$ 的车。租：首付 $\$2000$，每月 $\$329$ 共 48 个月，费用 $\$500$。买：不付首付，贷款每月 $\$580$ 共 48 个月，之后卖掉（每年贬值 15%）。4 年里哪个花得少?

1 Lease: $\$329$ a month. Loan: $\$580$ a month.

> 租：每月 $\$329$。贷款：每月 $\$580$。

2 The lease is $\$251$ a month less, so leasing costs less.

> 租每月少 $\$251$，所以租更省钱。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么?

2. Forgetting to subtract the resale value

> **忘了减去转卖价值**

Buy a $\$28\,000$ car for cash (it loses 15% a year and is sold after 3 years), or lease: $\$2500$ down, $\$359$ a month for 36 months, $\$400$ in fees?

> 现金买一辆 $\$28\,000$ 的车（每年贬值 15%，3 年后卖掉），还是租：首付 $\$2500$，每月 $\$359$ 共 36 个月，费用 $\$400$?

1 Buy: $\$28\,000$

> 买：$\$28\,000$

2 Lease: $2500 + 359 \times 36 + 400 = \$15\,824$

> 租：$2500 + 359 \times 36 + 400 = \$15\,824$

3 Leasing is cheaper by $\$12\,176$.

> 租便宜 $\$12\,176$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么?

3. Taking the same amount off every year

每年都减同样的钱

A $\$24\,000$ car loses 20% of its value each year. What is it worth after 3 years?

一辆 $\$24\,000$ 的车每年贬值 20%。3 年后值多少?

1 Each year it loses $0.2 \times 24\,000 = 4800$.

每年少 $0.2 \times 24\,000 = 4800$。

2 $V = 24\,000 - 3(4800) = 9600$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么?

4. Multiplying by the rate instead of what is kept

乘的是贬值率而不是保留率

A $\$15\,000$ car loses 10% of its value each year. What is it worth after 2 years?

一辆 $\$15\,000$ 的车每年贬值 10%。2 年后值多少?

1 $V = 15\,000(0.1)^2$

2 $V = \$150$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么?

5. Counting the whole mortgage payment as a cost

把整笔房贷还款都算成本

Rent is $\$2000$ a month. Owning a similar home this year: mortgage payments $\$24\,000$ (of which $\$15\,000$ is interest), property tax $\$2000$, strata $\$3600$, maintenance $\$1200$, lost return on the down payment $\$2500$. Which costs less in money you never get back?

月租 $\$2000$。买一套类似的房子，今年要付：房贷还款 $\$24\,000$（其中利息 $\$15\,000$）、地税 $\$2000$、物业费 $\$3600$、维修 $\$1200$、首付的机会成本 $\$2500$。按拿不回来的钱算，哪个更省?

1 Own: $24\,000 + 2000 + 3600 + 1200 + 2500 = \$33\,300$

买：$24\,000 + 2000 + 3600 + 1200 + 2500 = \$33\,300$

2 Rent: $2000 \times 12 = \$24\,000$

租：$2000 \times 12 = \$24\,000$

3 Renting is $\$9300$ cheaper.

租房便宜 $\$9300$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么?

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 2 —** A lease payment is low because you never own the car. Buying hands you a car worth thousands at the end, and the monthly numbers ignore that.

   > 租的月付低，是因为车永远不归你。买车到最后手里有一辆值好几千元的车，只看月供就把这一点漏掉了。

   Lease: $2000 + 329 \times 48 + 500 = \$18\,292$. Buy: $580 \times 48 = 27\,840$, minus resale $26\,000(0.85)^4 \approx 13\,572.16$, gives $\$14\,267.84$. Buying is about $\$4024.16$ cheaper.

   > 租：$2000 + 329 \times 48 + 500 = \$18\,292$。买：$580 \times 48 = 27\,840$，减去转卖价值 $26\,000(0.85)^4 \approx 13\,572.16$，得 $\$14\,267.84$。买约便宜 $\$4024.16$。

2. **line · 第 1 —** After 3 years the buyer still owns a car and sells it. That money comes back, so it must be taken off the buy side.

   > 3 年后买车的人手里还有车，可以卖掉。这笔钱会回来，要从「买」那一栏减掉。

   Resale $28\,000(0.85)^3 \approx \$17\,195.50$, so the net cost of buying is $\$10\,804.50$. Buying is cheaper by $\$5019.50$.

   > 转卖价值 $28\,000(0.85)^3 \approx \$17\,195.50$，所以买的净成本是 $\$10\,804.50$。买便宜 $\$5019.50$。

3. **line · 第 1 —** The 20% is taken from the value AT THAT TIME. After year 1 the car is worth less, so 20% of it is less than $\$4800$.

   > 20% 是按「当时」的价值算的。第一年后车已经不值那么多了，它的 20% 就不到 $\$4800$。

   It keeps 80% each year: $V = 24\,000(0.8)^3 = \$12\,288$.

   > 每年保留 80%：$V = 24\,000(0.8)^3 = \$12\,288$。

4. **line · 第 1 —** Losing 10% means keeping 90%. Multiplying by 0.1 keeps only 10% each year, which is why the answer is absurdly small.

   > 贬值 10% 就是保留 90%。乘 0.1 等于每年只留下 10%，所以答案小得离谱。

   $V = 15\,000(0.9)^2 = \$12\,150$.

   > $V = 15\,000(0.9)^2 = \$12\,150$。

5. **line · 第 1 —** Only the $\$15\,000$ of interest is gone. The other $\$9000$ pays down the loan, so it stays yours as equity in the home.

   > 真正花掉的只有 $\$15\,000$ 利息。另外 $\$9000$ 用来还本金，变成你在房子里的净值，仍然是你的。

   Own: $15\,000 + 2000 + 3600 + 1200 + 2500 = \$24\,300$. Renting is cheaper by only $\$300$, so a small rise in the home's value would tip it the other way.

   > 买：$15\,000 + 2000 + 3600 + 1200 + 2500 = \$24\,300$。租房只便宜 $\$300$，房价稍微涨一点就会反过来。