Simple trig equations

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** The interval (usually $0^\circ \le \theta < 360^\circ$) and the equation. Isolate the ratio: $\sin\theta$, $\cos\theta$ or $\tan\theta$ equals a number. If sine or cosine is outside $[-1, 1]$, stop: no solution.
2. 🎯 **Asked** ALL the angles in the interval, exact if the value is special, otherwise rounded as asked. Usually two; 0 and ± 1 give angles on the axes.
3. ✏️ **Picture it** A quick unit circle: mark where the ratio is positive or negative (CAST), then draw two terminal arms with the same reference angle.
4. 🧰 **Tool** Reference angle from the POSITIVE value (special triangle or inverse key). Then Q I: θ_R, Q II: $180^\circ - \theta_R$, Q III: $180^\circ + \theta_R$, Q IV: $360^\circ - \theta_R$.
5. ✅ **Check** Put each angle into the ORIGINAL equation (calculator in degrees). Count them: did you find both?

⭐ Must know

- Isolate the ratio first; $|\sin\theta|$ or $|\cos\theta| > 1$ → no solution.
- Reference angle from the POSITIVE value; the sign picks two quadrants (CAST).
- 0 and ± 1 are quadrantal: read them off the unit circle.

QI: θ_R QII: $180^\circ - \theta_R$ QIII: $180^\circ + \theta_R$ QIV: $360^\circ - \theta_R$

🔍 Watch out for

1. Stopping at the calculator answer
2. Taking the reference angle from the negative value
3. Choosing the wrong pair of quadrants
4. Dividing before moving the constant
5. Missing that the value is out of range

🌱 Small wins — tick as you go

- ○ I can solve a two-step linear equation, use $\sin^{-1}$ in degree mode, and say where x or y is negative. (5′)
- ○ I can ask the five starting questions for a trig equation. (5′)
- ○ I can follow, then finish, an isolate–reference angle–quadrants–check solution. (8′)
- ○ I can find BOTH angles on my own with at most two hints. (7′)
- ○ I can spot the five classic trig-equation mistakes. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Solve $3y + 2 = -4$.
A. $y = -\frac{2}{3}$ B. $y = -2$ C. $y = 2$ D. $y = -6$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x - 2) = x + 8 \rightarrow 3x - 6 = x + 8 \rightarrow 2x = 14 \rightarrow x = 7$.
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30 \rightarrow x = 6$.
3. Check by substituting: $3(7 - 2) = 15$ and $7 + 8 = 15$ ✓.

2. Solve $\sqrt{2}\,s + 1 = 0$ for s.
A. $-\sqrt{2}$ B. $\frac{1}{\sqrt{2}}$ C. $-\frac{1}{\sqrt{2}}$ D. $-1 - \sqrt{2}$

If you missed it — **Rearranging a formula**: Treat the other letters as numbers and undo operations in reverse order. $A = P(1 + r)$ for r: divide by P, then subtract 1: $r = \frac{A}{P} - 1$.

1. $y = a(x - p)^2 + q$ for a: subtract q, divide by $(x - p)^2$: $a = \frac{y - q}{(x - p)^2}$.
2. Substitute numbers first if the question gives them — fewer letters, fewer slips.

3. With the calculator in degrees, $\sin^{-1}(0.5) =$
A. $30°$ B. $60°$ C. $0.52°$ D. $45°$

If you missed it — **SOH CAH TOA and Pythagoras**: In a RIGHT triangle, from angle θ: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$. The hypotenuse is opposite the right angle; $a^2 + b^2 = c^2$. Calculator in DEGREE mode.

1. Legs 3 and 4: hypotenuse $\sqrt{9 + 16} = 5$; $\sin\theta = \frac{3}{5}$ for the angle opposite 3.
2. Find an angle: $\theta = \sin^{-1}(0.6) \approx 36.9°$.

4. In which two quadrants is the x-coordinate negative?
A. I and II B. I and IV C. III and IV D. II and III

If you missed it — **Reading a graph: points, intercepts, slope**: A point is (x, y): across first, then up. x-intercepts are where $y = 0$ (the graph crosses the horizontal axis); the y-intercept is where $x = 0$. Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

1. For $y = x^2 - 4$: y-intercept at $x = 0$ is -4; x-intercepts solve $x^2 - 4 = 0$: $x = \pm 2$.
2. Above the x-axis means $y > 0$; below means $y < 0$.

Answers: 1: B · 2: C · 3: A · 4: D

🔍 Common-errors sheet

1. Stopping at the calculator answer

Solve $\sin\theta = 0.6$ for $0^\circ \le \theta < 360^\circ$, to the nearest tenth.

1 $\theta = \sin^{-1} 0.6 \approx 36.9^\circ$

2 Answer: $\theta \approx 36.9^\circ$.

✍ Which line goes wrong first, and why?

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2. Taking the reference angle from the negative value

Solve $\cos\theta = -0.4$ for $0^\circ \le \theta < 360^\circ$, to the nearest tenth.

1 $\theta_R = \cos^{-1}(-0.4) \approx 113.6^\circ$

2 Q II: $180^\circ - 113.6^\circ = 66.4^\circ$; Q III: $180^\circ + 113.6^\circ = 293.6^\circ$.

✍ Which line goes wrong first, and why?

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3. Choosing the wrong pair of quadrants

Solve $\tan\theta = -1$ for $0^\circ \le \theta < 360^\circ$.

1 $\theta_R = 45^\circ$

2 Tangent is negative in Q II and Q III

3 $\theta = 135^\circ,\ 225^\circ$

✍ Which line goes wrong first, and why?

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4. Dividing before moving the constant

Solve $2\cos\theta - 1 = 0$ for $0° \le \theta < 360°$.

1 Divide by 2: $\cos\theta - 1 = 0$

2 $\cos\theta = 1$, so $\theta = 0°$.

✍ Which line goes wrong first, and why?

5. Missing that the value is out of range

Solve $4\sin\theta + 7 = 2$ for $0° \le \theta < 360°$.

1 $4\sin\theta = -5$

2 $\sin\theta = -\frac{4}{5}$

3 $\theta_R \approx 53.1°$, so $\theta \approx 233.1°,\ 306.9°$.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** The inverse key returns ONE angle. Sine is positive in Q II as well, so a second angle has the same sine. Q II: $180° - 36.87° = 143.13°$. Answers: $36.9°$ and $143.1°$.
2. **line 1 —** A reference angle is always between $0°$ and $90°$. Using the negative value gives an angle already in Q II, and the quadrant rules then move it twice. $\theta_R = \cos^{-1} 0.4 \approx 66.42°$. Q II: $113.6°$; Q III: $246.4°$.
3. **line 2 —** In Q III both x and y are negative, so $\tan = \frac{y}{x}$ is positive (the T in CAST). Tangent is negative in Q II and Q IV. Q II: $180° - 45° = 135°$; Q IV: $360° - 45° = 315°$.
4. **line 1 —** Dividing by 2 must divide EVERY term: $\frac{2\cos\theta - 1}{2} = \cos\theta - \frac{1}{2}$. Moving the constant first avoids the trap. $2\cos\theta = 1$, $\cos\theta = \frac{1}{2}$, $\theta_R = 60°$; cosine positive in Q I and Q IV: $60°$, $300°$.
5. **line 2 —** Dividing by 4 gives $-\frac{5}{4}$, not $-\frac{4}{5}$. The flipped fraction looked "normal", so the impossible value was never noticed. $\sin\theta = -1.25 < -1$. Sine is never below -1: no solution.