Angles in standard position

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Read the angle and its sign: positive turns counterclockwise, negative turns clockwise. Is it bigger than 360° or below 0°?
2. 🎯 **Asked** What is wanted: the quadrant, the reference angle (acute, measured to the x-axis), or coterminal angles? All answers in degrees.
3. ✏️ **Picture it** A 3-second sketch: axes marked 90°, 180°, 270°, 360°, the terminal arm, and a small arc from the arm to the x-axis.
4. 🧰 **Tool** First add or subtract 360° until the angle is between 0° and 360°. Then find the quadrant, then use that quadrant's rule: θ, $180^\circ - \theta$, $\theta - 180^\circ$ or $360^\circ - \theta$.
5. ✅ **Check** The reference angle must be between 0° and 90° and match the sketch. Two angles are coterminal only if their difference is a multiple of 360°.

⭐ Must know

- Start on the positive x-axis; counterclockwise is $+$, clockwise is $-$.
- Reference angle: acute angle to the x-axis. QII $180^\circ - \theta$, QIII $\theta - 180^\circ$, QIV $360^\circ - \theta$.
- Coterminal: $\theta \pm 360^\circ n$.

$$\theta_{\text{coterminal}} = \theta + 360^\circ n,\ n \in \mathbb{Z}$$

$$\theta_R = 180^\circ - \theta,\ \ \theta - 180^\circ,\ \ 360^\circ - \theta$$

🔍 Watch out for

1. Measuring the reference angle to the y-axis
2. Using another quadrant's rule
3. Thinking a negative angle shares the arm of the positive one
4. Turning a negative angle the wrong way
5. Taking off only one full turn

🌱 Small wins — tick as you go

- ○ I can name the quadrant of a point and add or subtract 360 with negative numbers. (5′)
- ○ I can ask the five starting questions for any angle, big or negative. (5′)
- ○ I can follow, then finish, a reduce–sketch–reference-angle solution. (8′)
- ○ I can bring a negative angle into one turn and find its quadrant and reference angle with at most two hints. (6′)
- ○ I can spot the five classic angle mistakes in someone else's work. (8′)
- ○ I can do the Level 1–3 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. In which quadrant is the point $(-3, -5)$?
A. I B. II C. III D. IV

If you missed it — **Reading a graph: points, intercepts, slope**: A point is (x, y): across first, then up. x-intercepts are where $y = 0$ (the graph crosses the horizontal axis); the y-intercept is where $x = 0$. Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

1. For $y = x^2 - 4$: y-intercept at $x = 0$ is -4; x-intercepts solve $x^2 - 4 = 0$: $x = \pm 2$.
2. Above the x-axis means $y > 0$; below means $y < 0$.

2. Work out $-480 + 360$.
A. -840 B. -120 C. 120 D. 840

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

3. Two angles on a straight line add to $180°$. One is $35°$. Solve $x + 35 = 180$.
A. $55°$ B. $145°$ C. $215°$ D. $135°$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x - 2) = x + 8 \rightarrow 3x - 6 = x + 8 \rightarrow 2x = 14 \rightarrow x = 7$.
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30 \rightarrow x = 6$.
3. Check by substituting: $3(7 - 2) = 15$ and $7 + 8 = 15$ ✓.

4. Work out $-150 + 360$.
A. -510 B. 510 C. 210 D. -210

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Answers: 1: C · 2: B · 3: B · 4: C

🔍 Common-errors sheet

1. Measuring the reference angle to the y-axis

Find the reference angle of 240°.

1 240° is in quadrant III.

2 From the arm to the nearest axis, 270°: $270^\circ - 240^\circ = 30^\circ$.

✍ Which line goes wrong first, and why?

2. Using another quadrant's rule

Find the reference angle of 320°.

1 320° is in quadrant IV.

2 Reference angle $= 320^\circ - 180^\circ = 140^\circ$.

✍ Which line goes wrong first, and why?

3. Thinking a negative angle shares the arm of the positive one

Give the angle between 0° and 360° that is coterminal with -70°.

1 Coterminal means the same size, so drop the minus sign: 70°.

2 Answer: 70°.

✍ Which line goes wrong first, and why?

4. Turning a negative angle the wrong way

State the quadrant and reference angle of -120°.

1 120° is between 90° and 180°, so -120° is in quadrant II.

2 Reference angle $180^\circ - 120^\circ = 60^\circ$.

✍ Which line goes wrong first, and why?

5. Taking off only one full turn

For $820°$, find the quadrant and the reference angle.

1 $820° - 360° = 460°$.

2 $460°$ is past $180°$, so quadrant III, and the reference angle is $460° - 180° = 280°$.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** The arm looks closer to the y-axis, so it feels natural to measure there. But a reference angle is always measured to the x-axis. Measure back to the negative x-axis at $180°$: $240° - 180° = 60°$.
2. **line 2 —** The rules were learned as a list, and the wrong line of the list was picked without a sketch. $140°$ is not even acute, which is a warning sign. Quadrant IV is $360° - \theta$: $360° - 320° = 40°$.
3. **line 1 —** The minus sign is a direction, not decoration. $-70°$ turns clockwise into quadrant IV; $70°$ turns the other way into quadrant I. Add a full turn: $-70° + 360° = 290°$.
4. **line 1 —** The quadrant was read as if the angle turned counterclockwise. Clockwise, $-90°$ is straight down and $-180°$ is the negative x-axis, so $-120°$ lands in quadrant III. $-120° + 360° = 240°$: quadrant III, reference angle $240° - 180° = 60°$. (The $60°$ happened to be right; the quadrant was not.)
5. **line 2 —** The first subtraction was fine, but $460°$ is still more than a full turn. The quadrant rules only work between $0°$ and $360°$; a $280°$ "reference angle" is the giveaway. Take off another turn: $460° - 360° = 100°$, quadrant II, reference angle $180° - 100° = 80°$.