Angles in standard position

标准位置角

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Read the angle and its sign: positive turns counterclockwise, negative turns clockwise. Is it bigger than 360° or below 0°?
   > 读出这个角和它的符号：正角逆时针转，负角顺时针转。它大于 360° 吗？小于 0° 吗？
2. 🎯 **Asked · 求什么** What is wanted: the quadrant, the reference angle (acute, measured to the x-axis), or coterminal angles? All answers in degrees.
   > 要求的是什么：所在象限、参考角（锐角，量到 x 轴），还是同终边角？答案都用度。
3. ✏️ **Picture it · 画出来** A 3-second sketch: axes marked 90°, 180°, 270°, 360°, the terminal arm, and a small arc from the arm to the x-axis.
   > 花 3 秒画草图：坐标轴上标出 90°、180°、270°、360°，画出终边，再画一小段从终边到 x 轴的弧。
4. 🧰 **Tool · 用什么工具** First add or subtract 360° until the angle is between 0° and 360°. Then find the quadrant, then use that quadrant's rule: θ, $180^\circ - \theta$, $\theta - 180^\circ$ or $360^\circ - \theta$.
   > 先加或减 360°，直到角落在 0° 到 360° 之间；再定象限；最后用该象限的公式：θ、$180^\circ - \theta$、$\theta - 180^\circ$ 或 $360^\circ - \theta$。
5. ✅ **Check · 检验** The reference angle must be between 0° and 90° and match the sketch. Two angles are coterminal only if their difference is a multiple of 360°.
   > 参考角必须在 0° 到 90° 之间，并且和草图对得上。两个角只有在差是 360° 的整数倍时才同终边。

⭐ Must know

必须记住

- Start on the positive x-axis; counterclockwise is $+$, clockwise is $-$.
  > 从 x 轴正半轴出发；逆时针为正，顺时针为负。
- Reference angle: acute angle to the x-axis. QII $180^\circ - \theta$, QIII $\theta - 180^\circ$, QIV $360^\circ - \theta$.
  > 参考角：终边与 x 轴所成锐角。第二象限 $180^\circ - \theta$，第三 $\theta - 180^\circ$，第四 $360^\circ - \theta$。
- Coterminal: $\theta \pm 360^\circ n$.
  > 终边相同的角：$\theta \pm 360^\circ n$。

$$\theta_{\text{coterminal}} = \theta + 360^\circ n,\ n \in \mathbb{Z}$$

$$\theta_R = 180^\circ - \theta,\ \ \theta - 180^\circ,\ \ 360^\circ - \theta$$

🔍 Watch out for

当心

1. Measuring the reference angle to the y-axis
   > 把参考角量到了 y 轴
2. Using another quadrant's rule
   > 用错了象限的公式
3. Thinking a negative angle shares the arm of the positive one
   > 以为负角和对应的正角终边相同
4. Turning a negative angle the wrong way
   > 负角转错了方向
5. Taking off only one full turn
   > 只减了一整圈

🌱 Small wins — tick as you go

一小步一小步——做完打勾

- ○ I can name the quadrant of a point and add or subtract 360 with negative numbers.
  > 我能说出一个点在哪个象限，也会带负数加减 360。

  (5′)
- ○ I can ask the five starting questions for any angle, big or negative.
  > 不管角多大、是不是负角，我都能问出五个入手问题。

  (5′)
- ○ I can follow, then finish, a reduce–sketch–reference-angle solution.
  > 我能看懂并补全「化到一圈内—画草图—求参考角」的解法。

  (8′)
- ○ I can bring a negative angle into one turn and find its quadrant and reference angle with at most two hints.
  > 我能在最多两个提示下，把负角化到一圈以内，求出象限和参考角。

  (6′)
- ○ I can spot the five classic angle mistakes in someone else's work.
  > 我能在别人的解答里找出五个关于角的经典错误。

  (8′)
- ○ I can do the Level 1–3 practice questions.
  > 我能做出 1–3 级练习题。

  (10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. In which quadrant is the point $(-3, -5)$?

> 点 $(-3, -5)$ 在第几象限?

A. I B. II C. III D. IV

If you missed it · 如果做错了 — **Reading a graph: points, intercepts, slope**

> **读图：点、截距、斜率**

: A point is (x, y): across first, then up. x-intercepts are where $y = 0$ (the graph crosses the horizontal axis); the y-intercept is where $x = 0$. Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

> 点写作 (x, y)：先横后纵。x 截距是 $y = 0$ 的地方（图象穿过横轴处）；y 截距是 $x = 0$ 的地方。斜率 $= \frac{\text{纵向变化}}{\text{横向变化}} = \frac{y_2 - y_1}{x_2 - x_1}$。

1. For $y = x^2 - 4$: y-intercept at $x = 0$ is -4; x-intercepts solve $x^2 - 4 = 0$: $x = \pm 2$.
   > $y = x^2 - 4$：令 $x = 0$ 得 y 截距 -4；令 $y = 0$ 解 $x^2 - 4 = 0$ 得 x 截距 $x = \pm 2$。
2. Above the x-axis means $y > 0$; below means $y < 0$.
   > 在 x 轴上方表示 $y > 0$；下方表示 $y < 0$。

2. Work out $-480 + 360$.

> 计算 $-480 + 360$。

A. -840 B. -120 C. 120 D. 840

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

> 减去负数等于加上它：$5 - (-3) = 5 + 3 = 8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2 = 16$，但 $-4^2 = -16$（先算平方）。

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
   > $-7 - (-2)$：把「减负」改成「加」：$-7 + 2 = -5$。
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
   > $(-3)(-5) = 15$（同号），$(-3)(5) = -15$（异号）。
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.
   > 把 $x = -3$ 代入 x^2：先加括号，$(-3)^2 = 9$。

3. Two angles on a straight line add to $180°$. One is $35°$. Solve $x + 35 = 180$.

> 一条直线上的两个角加起来是 $180°$，其中一个是 $35°$。解 $x + 35 = 180$。

A. $55°$ B. $145°$ C. $215°$ D. $135°$

If you missed it · 如果做错了 — **Solving a linear equation**

> **解一元一次方程**

: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

> 两边做同样的事。先去括号，把含 x 的项移到一边、数字移到另一边，最后除以系数。有分数时先把每一项都乘以公分母。

1. $3(x - 2) = x + 8 \to 3x - 6 = x + 8 \to 2x = 14 \to x = 7$.
   > $3(x - 2) = x + 8 \to 3x - 6 = x + 8 \to 2x = 14 \to x = 7$。
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30 \to x = 6$.
   > $\frac{x}{2} + \frac{x}{3} = 5$：每项乘 6 → $3x + 2x = 30 \to x = 6$。
3. Check by substituting: $3(7 - 2) = 15$ and $7 + 8 = 15$ ✓.
   > 代回检验：$3(7 - 2) = 15$，$7 + 8 = 15$ ✓。

4. Work out $-150 + 360$.

> 计算 $-150 + 360$。

A. -510 B. 510 C. 210 D. -210

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

> 减去负数等于加上它：$5 - (-3) = 5 + 3 = 8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2 = 16$，但 $-4^2 = -16$（先算平方）。

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
   > $-7 - (-2)$：把「减负」改成「加」：$-7 + 2 = -5$。
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
   > $(-3)(-5) = 15$（同号），$(-3)(5) = -15$（异号）。
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.
   > 把 $x = -3$ 代入 x^2：先加括号，$(-3)^2 = 9$。

Answers · 答案: 1: C · 2: B · 3: B · 4: C

🔍 Common-errors sheet

常见错误单

1. Measuring the reference angle to the y-axis

> **把参考角量到了 y 轴**

Find the reference angle of $240°$.

> 求 $240°$ 的参考角。

1 $240°$ is in quadrant III.

> $240°$ 在第三象限。

2 From the arm to the nearest axis, $270°$: $270° - 240° = 30°$.

> 从终边量到最近的轴 $270°$：$270° - 240° = 30°$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Using another quadrant's rule

> **用错了象限的公式**

Find the reference angle of $320°$.

> 求 $320°$ 的参考角。

1 $320°$ is in quadrant IV.

> $320°$ 在第四象限。

2 Reference angle $= 320° - 180° = 140°$.

> 参考角 $= 320° - 180° = 140°$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Thinking a negative angle shares the arm of the positive one

> **以为负角和对应的正角终边相同**

Give the angle between $0°$ and $360°$ that is coterminal with $-70°$.

> 写出 $0°$ 到 $360°$ 之间与 $-70°$ 终边相同的角。

1 Coterminal means the same size, so drop the minus sign: $70°$.

> 同终边就是大小一样，所以去掉负号：$70°$。

2 Answer: $70°$.

> 答案：$70°$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Turning a negative angle the wrong way

> **负角转错了方向**

State the quadrant and reference angle of -120°.

> 写出 -120° 所在象限和参考角。

1 120° is between 90° and 180°, so -120° is in quadrant II.

> 120° 在 90° 和 180° 之间，所以 -120° 在第二象限。

2 Reference angle $180^\circ - 120^\circ = 60^\circ$.

> 参考角 $180^\circ - 120^\circ = 60^\circ$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Taking off only one full turn

> **只减了一整圈**

For 820°, find the quadrant and the reference angle.

> 对 820°，求所在象限和参考角。

1 $820^\circ - 360^\circ = 460^\circ$.

> $820^\circ - 360^\circ = 460^\circ$。

2 460° is past 180°, so quadrant III, and the reference angle is $460^\circ - 180^\circ = 280^\circ$.

> 460° 超过了 180°，所以在第三象限，参考角是 $460^\circ - 180^\circ = 280^\circ$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 2 —** The arm looks closer to the y-axis, so it feels natural to measure there. But a reference angle is always measured to the x-axis.

   > 终边看起来离 y 轴更近，所以很自然就量到那里去了。但参考角永远量到 x 轴。

   Measure back to the negative x-axis at 180°: $240^\circ - 180^\circ = 60^\circ$.

   > 量回到 x 轴负半轴（180°）：$240^\circ - 180^\circ = 60^\circ$。

2. **line · 第 2 —** The rules were learned as a list, and the wrong line of the list was picked without a sketch. 140° is not even acute, which is a warning sign.

   > 公式是当成一张表背下来的，没画草图就挑错了一行。140° 连锐角都不是，这本身就是警报。

   Quadrant IV is $360^\circ - \theta$: $360^\circ - 320^\circ = 40^\circ$.

   > 第四象限用 $360^\circ - \theta$：$360^\circ - 320^\circ = 40^\circ$。

3. **line · 第 1 —** The minus sign is a direction, not decoration. -70° turns clockwise into quadrant IV; 70° turns the other way into quadrant I.

   > 负号表示方向，不是装饰。-70° 顺时针转到第四象限；70° 逆时针转到第一象限。

   Add a full turn: $-70^\circ + 360^\circ = 290^\circ$.

   > 加一整圈：$-70^\circ + 360^\circ = 290^\circ$。

4. **line · 第 1 —** The quadrant was read as if the angle turned counterclockwise. Clockwise, -90° is straight down and -180° is the negative x-axis, so -120° lands in quadrant III.

   > 把它当成逆时针转来读象限了。顺时针转时，-90° 指向正下方，-180° 是 x 轴负半轴，所以 -120° 落在第三象限。

   $-120^\circ + 360^\circ = 240^\circ$: quadrant III, reference angle $240^\circ - 180^\circ = 60^\circ$. (The 60° happened to be right; the quadrant was not.)

   > $-120^\circ + 360^\circ = 240^\circ$：第三象限，参考角 $240^\circ - 180^\circ = 60^\circ$。（$60^\circ$ 碰巧对了，象限却错了。）

5. **line · 第 2 —** The first subtraction was fine, but 460° is still more than a full turn. The quadrant rules only work between 0° and 360°; a 280° "reference angle" is the giveaway.

   > 第一步减法没错，但 460° 仍然超过一整圈。象限公式只在 0° 到 360° 之间才能用；算出 280° 的「参考角」就露馅了。

   Take off another turn: $460^\circ - 360^\circ = 100^\circ$, quadrant II, reference angle $180^\circ - 100^\circ = 80^\circ$.

   > 再减一圈：$460^\circ - 360^\circ = 100^\circ$，第二象限，参考角 $180^\circ - 100^\circ = 80^\circ$。