# The sine law

Thinking map — keep it next to you while you work

## 🧭 Five questions before you write

1. 🔎 **Given** Label the triangle: side $a$ across from angle $A$, and so on. Find a complete pair (a side AND its opposite angle). If two angles are known, the third is $180°$ minus them.
2. 🎯 **Asked** Which side or angle, and how to round. A side (cm, m, km) or an angle (degrees)?
3. ✏️ **Picture it** Sketch it roughly to scale. The biggest angle faces the longest side: use that to predict your answer before you calculate.
4. 🧰 **Tool** Unknown on top: $\frac{b}{\sin B} = \frac{a}{\sin A}$ for a side, $\frac{\sin B}{b} = \frac{\sin A}{a}$ for an angle. Calculator in degrees; keep full values until the end.
5. ✅ **Check** Do the sides come in the same order as their opposite angles? Do the angles add to $180°$? Any negative length means radian mode.

## ⭐ Must know

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$; side $a$ is opposite angle $A$.
- Needs a complete pair (side + opposite angle): AAS, ASA, SSA. SAS or SSS → cosine law.
- Unknown on top: sides on top for a side, sines on top for an angle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

## 🔍 Watch out for

1. Pairing a side with the wrong angle
2. Ratio upside down when finding a side
3. Using a side before finding its angle
4. Calculator in radian mode
5. Rounding the sine too early

## 🌱 Small wins — tick as you go

- ○ I can find a third angle, use $\sin$ and $\sin^{-1}$ in degree mode, and solve a proportion. (5′)
- ○ I can label a triangle and find its complete side–angle pair before writing anything else. (5′)
- ○ I can follow, then finish, a pair–proportion–sense-check solution for a side. (8′)
- ○ I can find a side on my own when the pair is hidden (ASA), with at most two hints. (7′)
- ○ I can spot the five classic sine-law mistakes. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

# Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

**1.** With the calculator in degrees, $\sin 40^\circ$ to three decimals is:
A. $0.643$ B. $0.745$ C. $0.766$ D. $0.839$

If you missed it — **SOH CAH TOA and Pythagoras**: In a RIGHT triangle, from angle $\theta$: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$. The hypotenuse is opposite the right angle; $a^2 + b^2 = c^2$. Calculator in DEGREE mode.

1. Legs 3 and 4: hypotenuse $\sqrt{9+16} = 5$; $\sin\theta = \frac{3}{5}$ for the angle opposite 3.
2. Find an angle: $\theta = \sin^{-1}(0.6) \approx 36.9^\circ$.

**2.** With the calculator in degrees, $\sin^{-1}(0.8)$ to the nearest tenth is:
A. $36.9^\circ$ B. $0.9^\circ$ C. $38.7^\circ$ D. $53.1^\circ$

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**3.** A triangle has angles $47^\circ$ and $68^\circ$. What is the third angle?
A. $115^\circ$ B. $75^\circ$ C. $65^\circ$ D. $245^\circ$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect $x$ on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x-2) = x+8 \to 3x-6 = x+8 \to 2x = 14 \to x = 7$.
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30 \to x = 6$.
3. Check by substituting: $3(7-2) = 15$ and $7+8 = 15$ ✓.

**4.** Solve $\dfrac{x}{0.5} = \dfrac{12}{0.8}$.
A. $19.2$ B. $7.5$ C. $3.3$ D. $0.03$

If you missed it — **Rearranging a formula**: Treat the other letters as numbers and undo operations in reverse order. $A = P(1+r)$ for $r$: divide by $P$, then subtract 1: $r = \frac{A}{P} - 1$.

1. $y = a(x-p)^2 + q$ for $a$: subtract $q$, divide by $(x-p)^2$: $a = \frac{y-q}{(x-p)^2}$.
2. Substitute numbers first if the question gives them — fewer letters, fewer slips.

Answers: 1: A · 2: D · 3: C · 4: B

# 🔍 Common-errors sheet

**1. Pairing a side with the wrong angle**

In $\triangle ABC$, $\angle A = 45°$, $\angle B = 60°$ and $a = 10$. Find $b$ to the nearest tenth.

1 $\dfrac{b}{\sin 45°} = \dfrac{10}{\sin 60°}$

2 $b \approx 8.2$

✍ Which line goes wrong first, and why?

**2. Ratio upside down when finding a side**

In $\triangle ABC$, $\angle A = 30°$, $\angle C = 100°$ and $c = 18$. Find $a$ to the nearest tenth.

1 $\dfrac{a}{\sin 30°} = \dfrac{18}{\sin 100°}$

2 $a = \dfrac{18 \sin 100°}{\sin 30°} \approx 35.5$

✍ Which line goes wrong first, and why?

**3. Using a side before finding its angle**

In $\triangle ABC$, $\angle A = 35°$, $\angle B = 80°$ and $c = 12$. Find $a$ to the nearest tenth.

1 $\dfrac{a}{\sin 35°} = \dfrac{12}{\sin 80°}$

2 $a \approx 7.0$

✍ Which line goes wrong first, and why?

**4. Calculator in radian mode**

In $\triangle ABC$, $\angle A = 50°$, $\angle B = 70°$ and $a = 8$. Find $b$ to the nearest tenth.

1 $b = \dfrac{8 \sin 70°}{\sin 50°}$

2 Calculator: $\dfrac{8(0.774)}{-0.262} \approx -23.6$

✍ Which line goes wrong first, and why?

**5. Rounding the sine too early**

In $\triangle ABC$, $a = 14$, $b = 10$ and $\angle A = 65°$. Find $\angle B$ to the nearest tenth.

**1** $\dfrac{\sin B}{10} = \dfrac{\sin 65°}{14}$

**2** $\sin B = \dfrac{10 \sin 65°}{14} \approx 0.65$

**3** $B = \sin^{-1}(0.65) \approx 40.5°$

✍ Which line goes wrong first, and why?

**Key (fold this part back)**

1. **line 1 —** Each side goes with the angle ACROSS from it. $b$ belongs with $B = 60°$, and $a = 10$ with $A = 45°$. $\dfrac{b}{\sin 60°} = \dfrac{10}{\sin 45°} \Rightarrow b \approx 12.2$. Check: bigger angle, bigger side ✓.
2. **line 2 —** To get $a$ alone, multiply both sides by $\sin 30°$: the angle under $a$ moves to the top. The answer $35.5$ is also too big: $30°$ is the smallest angle. $a = \dfrac{18 \sin 30°}{\sin 100°} \approx 9.1$.
3. **line 1 —** The side $12$ is $c$, opposite $C$, which was never found. It got paired with the angle that happened to be given. $C = 180° - 35° - 80° = 65°$; $a = \dfrac{12 \sin 35°}{\sin 65°} \approx 7.6$.
4. **line 2 —** The set-up is right, but the calculator read 70 and 50 as radians. Sines of angles between $0°$ and $180°$ are always positive, so a negative value is the warning sign. In degrees: $\dfrac{8(0.9397)}{0.7660} \approx 9.8$.
5. **line 2 —** Rounding $0.6474$ to $0.65$ looks harmless, but $\sin^{-1}$ magnifies it: the angle moves by $0.2°$, which changes the tenths digit. Keep the full value: $\sin B \approx 0.647363$, so $B \approx 40.3°$. (Only one triangle: $b < a$, so $B < A$.)