# The sine law

## 正弦定理

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

### 🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Label the triangle: side $a$ across from angle $A$, and so on. Find a complete pair (a side AND its opposite angle). If two angles are known, the third is $180^\circ$ minus them.

   > 先标三角形：边 $a$ 在角 $A$ 的对面，依此类推。找一组完整的「边—对角」。若已知两个角，第三个角是 $180^\circ$ 减去它们。

2. 🎯 **Asked · 求什么** Which side or angle, and how to round. A side (cm, m, km) or an angle (degrees)?

   > 求哪条边或哪个角？保留几位？求的是边（cm、m、km）还是角（度）？

3. ✏️ **Picture it · 画出来** Sketch it roughly to scale. The biggest angle faces the longest side: use that to predict your answer before you calculate.

   > 大致按比例画草图。最大的角对着最长的边：计算之前先用它预估答案。

4. 🧰 **Tool · 用什么工具** Unknown on top: $\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$ for a side, $\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$ for an angle. Calculator in degrees; keep full values until the end.

   > 未知量放分子：求边用 $\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$，求角用 $\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$。计算器用角度制，中间值全部保留到最后。

5. ✅ **Check · 检验** Do the sides come in the same order as their opposite angles? Do the angles add to $180^\circ$? Any negative length means radian mode.

   > 边的大小顺序和对角的顺序一致吗？三角之和是 $180^\circ$ 吗？出现负的边长，说明计算器在弧度制。

### ⭐ Must know

必须记住

- $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$; side $a$ is opposite angle $A$.

  > $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$；边 $a$ 对角 $A$。

- Needs a complete pair (side + opposite angle): AAS, ASA, SSA. SAS or SSS → cosine law.

  > 需要一组完整的「边＋对角」：AAS、ASA、SSA 可用。SAS、SSS → 用余弦定理。

- Unknown on top: sides on top for a side, sines on top for an angle.

  > 未知量放分子：求边时边在上，求角时正弦在上。

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

### 🔍 Watch out for

当心

1. Pairing a side with the wrong angle
   > 边和角配错了
2. Ratio upside down when finding a side
   > 求边时比例倒过来了
3. Using a side before finding its angle
   > 还没求出对角就用了这条边
4. Calculator in radian mode
   > 计算器在弧度制
5. Rounding the sine too early
   > 正弦值过早取整

## 🌱 Small wins — tick as you go

一小步一小步——做完打勾

- ○ I can find a third angle, use $\sin$ and $\sin^{-1}$ in degree mode, and solve a proportion.
  > 我会求第三个角，会在角度制下用 $\sin$ 和 $\sin^{-1}$，会解比例式。

  (5′)
- ○ I can label a triangle and find its complete side–angle pair before writing anything else.
  > 动笔前，我能先标好三角形，找出完整的「边—对角」。

  (5′)
- ○ I can follow, then finish, a pair–proportion–sense-check solution for a side.
  > 我能看懂并补全求边的「找边角对—列比例—合理性检验」解法。

  (8′)
- ○ I can find a side on my own when the pair is hidden (ASA), with at most two hints.
  > 当「边—对角」藏起来时（ASA），我能在最多两个提示下独立求边。

  (7′)
- ○ I can spot the five classic sine-law mistakes.
  > 我能找出正弦定理的五个经典错误。

  (8′)
- ○ I can do the Level 1–2 practice questions.
  > 我能做出 1–2 级练习题。

  (10′)

# Prerequisite repair sheet

## 先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

**1.** With the calculator in degrees, $\sin 40^\circ$ to three decimals is:

> 计算器在角度制下，$\sin 40^\circ$ 保留三位小数是：

A. $0.643$ B. $0.745$ C. $0.766$ D. $0.839$

If you missed it · 如果做错了 — **SOH CAH TOA and Pythagoras**

> **直角三角形：SOH CAH TOA 与勾股定理**

: In a RIGHT triangle, from angle $\theta$: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$. The hypotenuse is opposite the right angle; $a^2 + b^2 = c^2$. Calculator in DEGREE mode.

> 在直角三角形中，从角 $\theta$ 看：$\sin\theta = \frac{\text{对边}}{\text{斜边}}$，$\cos\theta = \frac{\text{邻边}}{\text{斜边}}$，$\tan\theta = \frac{\text{对边}}{\text{邻边}}$。斜边对着直角；$a^2 + b^2 = c^2$。计算器设为角度（DEG）模式。

1. Legs 3 and 4: hypotenuse $\sqrt{9+16} = 5$; $\sin\theta = \frac{3}{5}$ for the angle opposite 3.
   > 两直角边 3 和 4：斜边 $\sqrt{9+16} = 5$；对着 3 的角 $\sin\theta = \frac{3}{5}$。
2. Find an angle: $\theta = \sin^{-1}(0.6) \approx 36.9^\circ$.
   > 求角：$\theta = \sin^{-1}(0.6) \approx 36.9^\circ$。

**2.** With the calculator in degrees, $\sin^{-1}(0.8)$ to the nearest tenth is:

> 计算器在角度制下，$\sin^{-1}(0.8)$ 精确到 0.1° 是：

A. $36.9^\circ$ B. $0.9^\circ$ C. $38.7^\circ$ D. $53.1^\circ$

If you missed it · 如果做错了 — **SOH CAH TOA and Pythagoras**

> **直角三角形：SOH CAH TOA 与勾股定理**

: In a RIGHT triangle, from angle $\theta$: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$. The hypotenuse is opposite the right angle; $a^2 + b^2 = c^2$. Calculator in DEGREE mode.

> 在直角三角形中，从角 $\theta$ 看：$\sin\theta = \frac{\text{对边}}{\text{斜边}}$，$\cos\theta = \frac{\text{邻边}}{\text{斜边}}$，$\tan\theta = \frac{\text{对边}}{\text{邻边}}$。斜边对着直角；$a^2 + b^2 = c^2$。计算器设为角度（DEG）模式。

1. Legs 3 and 4: hypotenuse $\sqrt{9+16} = 5$; $\sin\theta = \frac{3}{5}$ for the angle opposite 3.
   > 两直角边 3 和 4：斜边 $\sqrt{9+16} = 5$；对着 3 的角 $\sin\theta = \frac{3}{5}$。
2. Find an angle: $\theta = \sin^{-1}(0.6) \approx 36.9^\circ$.
   > 求角：$\theta = \sin^{-1}(0.6) \approx 36.9^\circ$。

**3.** A triangle has angles $47°$ and $68°$. What is the third angle?

> 三角形有两个角是 $47°$ 和 $68°$，第三个角是多少?

A. $115°$ B. $75°$ C. $65°$ D. $245°$

If you missed it · 如果做错了 — **Solving a linear equation**

> **解一元一次方程**

: Do the same thing to both sides. Clear brackets, collect $x$ on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

> 两边做同样的事。先去括号，把含 $x$ 的项移到一边、数字移到另一边，最后除以系数。有分数时先把每一项都乘以公分母。

1. $3(x-2)=x+8 \to 3x-6=x+8 \to 2x=14 \to x=7$.
   > $3(x-2)=x+8 \to 3x-6=x+8 \to 2x=14 \to x=7$。
2. $\frac{x}{2}+\frac{x}{3}=5$: multiply every term by 6 → $3x+2x=30 \to x=6$.
   > $\frac{x}{2}+\frac{x}{3}=5$：每项乘 6 → $3x+2x=30 \to x=6$。
3. Check by substituting: $3(7-2)=15$ and $7+8=15$ ✓.
   > 代回检验：$3(7-2)=15$，$7+8=15$ ✓。

**4.** Solve $\dfrac{x}{0.5}=\dfrac{12}{0.8}$.

> 解 $\dfrac{x}{0.5}=\dfrac{12}{0.8}$。

A. $19.2$ B. $7.5$ C. $3.3$ D. $0.03$

If you missed it · 如果做错了 — **Rearranging a formula**

> **公式变形（求某个字母）**

: Treat the other letters as numbers and undo operations in reverse order. $A=P(1+r)$ for $r$: divide by $P$, then subtract 1: $r=\frac{A}{P}-1$.

> 把其他字母当成数字，按相反顺序撤销运算。由 $A=P(1+r)$ 求 $r$：先除以 $P$，再减 1：$r=\frac{A}{P}-1$。

1. $y=a(x-p)^2+q$ for $a$: subtract $q$, divide by $(x-p)^2$: $a=\frac{y-q}{(x-p)^2}$.
   > 由 $y=a(x-p)^2+q$ 求 $a$：减 $q$，再除以 $(x-p)^2$：$a=\frac{y-q}{(x-p)^2}$。
2. Substitute numbers first if the question gives them — fewer letters, fewer slips.
   > 题目给了数字就先代入——字母越少，越不容易错。

Answers · 答案: 1: A · 2: D · 3: C · 4: B

# 🔍 Common-errors sheet

## 常见错误单

### 1. Pairing a side with the wrong angle

> **边和角配错了**

In $\triangle ABC$, $\angle A = 45^\circ$, $\angle B = 60^\circ$ and $a = 10$. Find $b$ to the nearest tenth.

> 在 $\triangle ABC$ 中，$\angle A = 45^\circ$，$\angle B = 60^\circ$，$a = 10$。求 $b$，精确到 0.1。

**1** $\dfrac{b}{\sin 45^\circ} = \dfrac{10}{\sin 60^\circ}$

**2** $b \approx 8.2$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

### 2. Ratio upside down when finding a side

> **求边时比例倒过来了**

In $\triangle ABC$, $\angle A = 30^\circ$, $\angle C = 100^\circ$ and $c = 18$. Find $a$ to the nearest tenth.

> 在 $\triangle ABC$ 中，$\angle A = 30^\circ$，$\angle C = 100^\circ$，$c = 18$。求 $a$，精确到 0.1。

**1** $\dfrac{a}{\sin 30^\circ} = \dfrac{18}{\sin 100^\circ}$

**2** $a = \dfrac{18 \sin 100^\circ}{\sin 30^\circ} \approx 35.5$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

### 3. Using a side before finding its angle

> **还没求出对角就用了这条边**

In $\triangle ABC$, $\angle A = 35^\circ$, $\angle B = 80^\circ$ and $c = 12$. Find $a$ to the nearest tenth.

> 在 $\triangle ABC$ 中，$\angle A = 35^\circ$，$\angle B = 80^\circ$，$c = 12$。求 $a$，精确到 0.1。

**1** $\dfrac{a}{\sin 35^\circ} = \dfrac{12}{\sin 80^\circ}$

**2** $a \approx 7.0$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

### 4. Calculator in radian mode

**计算器在弧度制**

In $\triangle ABC$, $\angle A = 50°$, $\angle B = 70°$ and $a = 8$. Find $b$ to the nearest tenth.

在 $\triangle ABC$ 中，$\angle A = 50°$，$\angle B = 70°$，$a = 8$。求 $b$，精确到 0.1。

**1** $b = \dfrac{8 \sin 70°}{\sin 50°}$

**2** Calculator: $\dfrac{8(0.774)}{-0.262} \approx -23.6$

计算器：$\dfrac{8(0.774)}{-0.262} \approx -23.6$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

### 5. Rounding the sine too early

**正弦值过早取整**

In $\triangle ABC$, $a = 14$, $b = 10$ and $\angle A = 65°$. Find $\angle B$ to the nearest tenth.

在 $\triangle ABC$ 中，$a = 14$，$b = 10$，$\angle A = 65°$。求 $\angle B$，精确到 0.1°。

**1** $\dfrac{\sin B}{10} = \dfrac{\sin 65°}{14}$

**2** $\sin B = \dfrac{10 \sin 65°}{14} \approx 0.65$

**3** $B = \sin^{-1}(0.65) \approx 40.5°$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

**Key (fold this part back) · 答案（可以折起来）**

1. **line · 第 1 —** Each side goes with the angle ACROSS from it. $b$ belongs with $B = 60°$, and $a = 10$ with $A = 45°$.

   > 每条边都和它「对面」的角配对。$b$ 配 $B = 60°$，$a = 10$ 配 $A = 45°$。

   $\dfrac{b}{\sin 60°} = \dfrac{10}{\sin 45°} \Rightarrow b \approx 12.2$. Check: bigger angle, bigger side ✓.

   > $\dfrac{b}{\sin 60°} = \dfrac{10}{\sin 45°} \Rightarrow b \approx 12.2$。检验：角大边也大 ✓。

2. **line · 第 2 —** To get $a$ alone, multiply both sides by $\sin 30°$: the angle under $a$ moves to the top. The answer $35.5$ is also too big: $30°$ is the smallest angle.

   > 要把 $a$ 单独留下，两边乘 $\sin 30°$：$a$ 下面的那个角要移到分子。而且 $35.5$ 太大了：$30°$ 是最小的角。

   $a = \dfrac{18 \sin 30°}{\sin 100°} \approx 9.1$.

   > $a = \dfrac{18 \sin 30°}{\sin 100°} \approx 9.1$。

3. **line · 第 1 —** The side $12$ is $c$, opposite $C$, which was never found. It got paired with the angle that happened to be given.

   > 边 $12$ 是 $c$，对角是 $C$，而 $C$ 还没求。结果它和恰好给出的那个角配在了一起。

   $C = 180° - 35° - 80° = 65°$; $a = \dfrac{12 \sin 35°}{\sin 65°} \approx 7.6$.

   > $C = 180° - 35° - 80° = 65°$；$a = \dfrac{12 \sin 35°}{\sin 65°} \approx 7.6$。

4. **line · 第 2 —** The set-up is right, but the calculator read 70 and 50 as radians. Sines of angles between $0°$ and $180°$ are always positive, so a negative value is the warning sign.

   > 式子是对的，但计算器把 70 和 50 当成了弧度。$0°$ 到 $180°$ 之间的角正弦都是正的，出现负数就是警报。

   In degrees: $\dfrac{8(0.9397)}{0.7660} \approx 9.8$.

   > 用角度制：$\dfrac{8(0.9397)}{0.7660} \approx 9.8$。

5. **line · 第 2 —** Rounding $0.6474$ to $0.65$ looks harmless, but $\sin^{-1}$ magnifies it: the angle moves by $0.2°$, which changes the tenths digit.

   > 把 $0.6474$ 取成 $0.65$ 看似无害，但 $\sin^{-1}$ 会放大误差：角度差了 $0.2°$，十分位就变了。

   Keep the full value: $\sin B \approx 0.647363$, so $B \approx 40.3°$. (Only one triangle: $b < a$, so $B < A$.)

   > 保留完整值：$\sin B \approx 0.647363$，所以 $B \approx 40.3°$。（只有一个三角形：$b < a$，所以 $B < A$。）