Trig ratios of any angle

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** What do you have: a point, or one ratio plus a clue (a quadrant, or the sign of another ratio)? Write the two of x, y, r you know, with signs.
2. 🎯 **Asked** Which ratios, exact or decimal? And θ in $0° \leq \theta < 360°$, rounded as asked.
3. ✏️ **Picture it** Sketch the axes, put the point in its quadrant, drop a line to the x-axis to make the reference triangle, and label x, y, r.
4. 🧰 **Tool** Missing piece from $x^2 + y^2 = r^2$ (sign from the quadrant, r always positive). Then $\sin = \frac{y}{r}$, $\cos = \frac{x}{r}$, $\tan = \frac{y}{x}$; reference angle from the positive ratio; place it: $180°-$, $180°+$ or $360°-$.
5. ✅ **Check** CAST: do the signs of your three ratios match the quadrant? Is θ really in that quadrant?

⭐ Must know

- $r = \sqrt{x^2 + y^2} > 0$; $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$.
- One ratio + quadrant: draw the triangle, find the third side, give x, y their quadrant signs.
- Angle: reference angle from the positive ratio, then place it in the quadrant.

$$r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r}, \quad \tan\theta = \frac{y}{x}$$

🔍 Watch out for

1. Wrong sign on the missing coordinate
2. Stopping at the calculator angle
3. Mixing up x and y in the ratios
4. Reading the sign clue as the wrong quadrant
5. Giving r a negative sign

🌱 Small wins — tick as you go

- ○ I can use SOH CAH TOA and Pythagoras, and name the quadrant of a point. (5′)
- ○ I can turn "one ratio plus a sign clue" into a quadrant and a labelled point. (5′)
- ○ I can follow, then finish, a point–triangle–ratios–angle solution. (8′)
- ○ I can find all three ratios and θ on my own with at most two hints. (7′)
- ○ I can spot the five classic sign and quadrant mistakes. (7′)
- ○ I can do the Level 1–2 practice questions. (8′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. A right triangle has legs 6 and 8. What is the sine of the angle opposite the side of length 6?
A. $\frac{3}{4}$ B. $\frac{4}{5}$ C. $\frac{3}{5}$ D. $\frac{4}{3}$

If you missed it — **SOH CAH TOA and Pythagoras**: In a RIGHT triangle, from angle θ: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$. The hypotenuse is opposite the right angle; $a^2 + b^2 = c^2$. Calculator in DEGREE mode.

1. Legs 3 and 4: hypotenuse $\sqrt{9+16} = 5$; $\sin\theta = \frac{3}{5}$ for the angle opposite 3.
2. Find an angle: $\theta = \sin^{-1}(0.6) \approx 36.9°$.

2. Find $\sqrt{9^2 + 12^2}$.
A. 21 B. 15 C. $\sqrt{21}$ D. 225

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.

3. Simplify $\dfrac{-6}{-8}$.
A. $\frac{3}{4}$ B. $-\frac{3}{4}$ C. $-\frac{4}{3}$ D. $\frac{4}{3}$

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

4. The point $(-4, 3)$ is in which quadrant?
A. I B. III C. IV D. II

If you missed it — **Reading a graph: points, intercepts, slope**: A point is (x, y): across first, then up. x-intercepts are where $y = 0$ (the graph crosses the horizontal axis); the y-intercept is where $x = 0$. Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

1. For $y = x^2 - 4$: y-intercept at $x = 0$ is -4; x-intercepts solve $x^2 - 4 = 0$: $x = \pm 2$.
2. Above the x-axis means $y > 0$; below means $y < 0$.

Answers: 1: C · 2: B · 3: A · 4: D

🔍 Common-errors sheet

1. Wrong sign on the missing coordinate

$\sin\theta = \frac{5}{13}$ and θ is in quadrant II. Find $\cos\theta$.

1 $y = 5,\ r = 13,\ x^2 = 169 - 25 = 144$

2 $x = 12$

3 $\cos\theta = \frac{12}{13}$

✍ Which line goes wrong first, and why?

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2. Stopping at the calculator angle

$P(8, -15)$ is on the terminal arm. Find θ, $0° \le \theta < 360°$, to the nearest tenth.

1 $\tan\theta = -\frac{15}{8}$

2 $\tan^{-1}\left(-\frac{15}{8}\right) \approx -61.9°$

3 $\theta = -61.9°$

✍ Which line goes wrong first, and why?

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3. Mixing up x and y in the ratios

$P(-9, 40)$ is on the terminal arm. Find $\sin\theta$.

1 $r = \sqrt{81 + 1600} = 41$

2 $\sin\theta = \frac{x}{r} = -\frac{9}{41}$

✍ Which line goes wrong first, and why?

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4. Reading the sign clue as the wrong quadrant

$\sin\theta = -0.6$ and $\tan\theta > 0$. Find θ, $0° \le \theta < 360°$, to the nearest tenth.

1 sin negative, so Q IV

2 Reference angle $\sin^{-1} 0.6 \approx 36.9°$

3 $\theta = 360° - 36.9° = 323.1°$

✍ Which line goes wrong first, and why?

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5. Giving r a negative sign

$P(-5, -12)$ is on the terminal arm. Find $\sin\theta$.

1 Both coordinates negative, so $r = -13$

2 $\sin\theta = \frac{-12}{-13} = \frac{12}{13}$

✍ Which line goes wrong first, and why?

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Key (fold this part back)

1. **line 2 —** A square root from a calculator is always positive, so the quadrant never got a say. In Q II the point is left of the y-axis. $x = \pm 12$; Q II needs $x < 0$, so $x = -12$ and $\cos\theta = -\frac{12}{13}$.
2. **line 3 —** The calculator answers in its own range, not the range the question set. Its output is a clue, not the answer. Q IV, reference angle $\tan^{-1}\frac{15}{8} \approx 61.9°$: $\theta = 360° - 61.9° = 298.1°$.
3. **line 2 —** SOH CAH TOA was learned with "opposite" and "adjacent", and the switch to x and y did not stick. Sine goes with the height y. $\sin\theta = \frac{y}{r} = \frac{40}{41}$ (and $\cos\theta = \frac{x}{r} = -\frac{9}{41}$).
4. **line 1 —** Only one clue was used. sin negative allows Q III or Q IV; the second clue, tan positive, is what picks between them. sin < 0 and tan > 0: Q III. $\theta = 180° + 36.9° = 216.9°$.
5. **line 1 —** It treats r like a coordinate. r is a distance from the origin, and a distance is never negative. $r = \sqrt{25 + 144} = 13$, so $\sin\theta = -\frac{12}{13}$ (Q III, sin negative ✓).