The ambiguous case (SSA)

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Label the known angle A, the side across from it a, and the other known side b (it touches A). Is A acute or obtuse?
2. 🎯 **Asked** How many triangles (0, 1 or 2)? If asked to solve, every missing angle and side for EACH triangle, rounded as stated.
3. ✏️ **Picture it** Draw A with side b to C, drop the height h from C, then swing side a from C like a gate and count where it hits the base.
4. 🧰 **Tool** $h = b\sin A$. Acute A: $a < h$ none, $a = h$ one (right), $h < a < b$ two, $a \geq b$ one. Then $\sin B = \dfrac{b\sin A}{a}$ and $B_2 = 180^\circ - B_1$.
5. ✅ **Check** For each triangle: angles add to 180°, and the longest side faces the largest angle. B_2 only counts if $A + B_2 < 180^\circ$.

⭐ Must know

- SSA ($\angle A$, a, b): height $h = b\sin A$.
- Acute A: $a < h$ none; $a = h$ one; $h < a < b$ TWO; $a \geq b$ one.
- Two triangles: $B_2 = 180^\circ - B_1$; finish each triangle separately.

$$h = b\sin A$$

$$B_2 = 180^\circ - B_1$$

🔍 Watch out for

1. Stopping at the first angle
2. Using the wrong side for the height
3. Comparing with only one length
4. Sine law ratio upside down
5. Reusing the first triangle's third angle

🌱 Small wins — tick as you go

- ○ I can find a side with $\sin$ in a right triangle and rearrange the sine law for $\sin B$. (5′)
- ○ I can label A, a, b and decide 0, 1 or 2 triangles from $h = b\sin A$. (6′)
- ○ I can follow, then finish, a two-triangle solution: h-test, B_1, B_2, then each C and c. (9′)
- ○ I can solve both triangles on my own with at most two hints. (8′)
- ○ I can spot the five classic SSA mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (9′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. A right triangle has hypotenuse 10 cm and an angle of $30°$. How long is the side opposite the $30°$ angle?
A. 8.7 cm B. 20 cm C. 5 cm D. 5.8 cm

If you missed it — **SOH CAH TOA and Pythagoras**: In a RIGHT triangle, from angle θ: $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$. The hypotenuse is opposite the right angle; $a^2 + b^2 = c^2$. Calculator in DEGREE mode.

1. Legs 3 and 4: hypotenuse $\sqrt{9+16} = 5$; $\sin\theta = \frac{3}{5}$ for the angle opposite 3.
2. Find an angle: $\theta = \sin^{-1}(0.6) \approx 36.9°$.

2. $\sin x = 0.5$ and x is acute. Find x.
A. $30°$ B. $60°$ C. $45°$ D. $0.5°$

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3. Solve $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ for $\sin B$.
A. $\sin B = \dfrac{a\sin A}{b}$ B. $\sin B = \dfrac{b\sin A}{a}$ C. $\sin B = \dfrac{ab}{\sin A}$ D. $\sin B = \dfrac{a}{b\sin A}$

If you missed it — **Rearranging a formula**: Treat the other letters as numbers and undo operations in reverse order. $A = P(1+r)$ for r: divide by P, then subtract 1: $r = \frac{A}{P} - 1$.

1. $y = a(x-p)^2 + q$ for a: subtract q, divide by $(x-p)^2$: $a = \frac{y-q}{(x-p)^2}$.
2. Substitute numbers first if the question gives them — fewer letters, fewer slips.

4. A triangle has angles $40°$ and $118°$. Find the third angle.
A. $32°$ B. $202°$ C. $78°$ D. $22°$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x-2) = x+8 \to 3x-6 = x+8 \to 2x = 14 \to x = 7$.
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30 \to x = 6$.
3. Check by substituting: $3(7-2) = 15$ and $7+8 = 15$ ✓.

Answers: 1: C · 2: A · 3: B · 4: D

🔍 Common-errors sheet

1. Stopping at the first angle

$\angle A = 38^\circ$, $b = 13$, $a = 10$. Find all possible values of $\angle B$ (nearest tenth).

1 $\sin B = \dfrac{13 \sin 38^\circ}{10} \approx 0.8004$

2 $\sin^{-1} 0.8004 \approx 53.2^\circ$

3 So $\angle B \approx 53.2^\circ$.

✍ Which line goes wrong first, and why?

2. Using the wrong side for the height

$\angle A = 40^\circ$, $b = 15$, $a = 9$. How many triangles?

1 $h = a \sin A = 9 \sin 40^\circ \approx 5.79$

2 $5.79 < 9 < 15$, so two triangles.

✍ Which line goes wrong first, and why?

3. Comparing with only one length

$\angle A = 30^\circ$, $b = 20$, $a = 13$. How many triangles?

1 $h = 20 \sin 30^\circ = 10$

2 $a = 13 > h$, so one triangle.

✍ Which line goes wrong first, and why?

4. Sine law ratio upside down

$\angle A = 45°$, $a = 12$, $b = 15$. Find all possible values of $\angle B$ (nearest tenth).

1 $\sin B = \dfrac{12 \sin 45°}{15} \approx 0.5657$

2 $B \approx 34.4°$

✍ Which line goes wrong first, and why?

5. Reusing the first triangle's third angle

$\angle A = 36°$, $b = 16$, $a = 11$. There are two triangles. Find $\angle C$ in each (nearest tenth).

1 $B_1 \approx 58.8°$, $C_1 = 180° - 36° - 58.8° = 85.2°$

2 $B_2 = 180° - 58.8° = 121.2°$

3 $C_2 = 85.2°$, same as before

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 3 —** The calculator shows one angle, so it feels finished. But $\sin(180° - 53.2°)$ has the same value, and nothing here rules that angle out. $h = 13 \sin 38° \approx 8.00 < 10 < 13$: two triangles. $B_2 = 126.8°$, and $38° + 126.8° < 180°$ ✓.
2. **line 1 —** "Angle A goes with side a" is right for the sine law but wrong for the height. The height drops from C, at the end of side b, so $h = b \sin A$. $h = 15 \sin 40° \approx 9.64 > 9$: side a is too short to reach the base. (Sine law agrees: $\sin B \approx 1.07 > 1$.)
3. **line 2 —** "Longer than the height" only proves a reaches the base. Whether it reaches it once or twice depends on a compared with b. $10 < 13 < 20$: two triangles ($B \approx 50.3°$ or $129.7°$).
4. **line 1 —** The two fractions were cross-multiplied from memory instead of written out. Each side goes with the sine of its OWN opposite angle. $\dfrac{\sin B}{15} = \dfrac{\sin 45°}{12}$, so $\sin B \approx 0.8839$: $B \approx 62.1°$ or $117.9°$ ($h \approx 10.61 < 12 < 15$).
5. **line 3 —** Only A, a and b are shared. Once B changes, the third angle and side change too: these are two different triangles. $C_2 = 180° - 36° - 121.2° = 22.8°$. Check: $36 + 121.2 + 22.8 = 180$ ✓.