Quadratic inequalities (sign analysis)

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Is one side 0? If not, move everything over first. Note the sign ($<$, $\leq$, $>$, $\geq$) and whether the x^2 coefficient is positive or negative.
2. 🎯 **Asked** Every x that makes it true. Usually one interval or two pieces joined by "or"; sometimes all real numbers, one point, or nothing.
3. ✏️ **Picture it** A tiny parabola through the zeros (up if $a > 0$, down if $a < 0$), or a number line with the zeros and $+/-$ in each region.
4. 🧰 **Tool** Factor (or use the formula) to get the zeros. Opens up: < 0 BETWEEN the zeros, > 0 OUTSIDE. Opens down: the other way round.
5. ✅ **Check** Test one number inside your answer and one outside in the ORIGINAL inequality. Square brackets only for $\leq$ or $\geq$.

⭐ Must know

- Everything to one side, find the zeros, then decide the sign between and outside them.
- Opens up: negative BETWEEN the zeros, positive OUTSIDE.
- $\leq$, $\geq$ include the zeros; outside answers are joined by "or" ($\cup$).

$$a(x-r)(x-s) < 0,\ a > 0,\ r < s \ \Rightarrow\ r < x < s$$

$$a(x-r)(x-s) > 0,\ a > 0 \ \Rightarrow\ x < r \text{ or } x > s$$

🔍 Watch out for

1. Picking the wrong region
2. Square-rooting an inequality
3. Factoring before comparing with zero
4. Multiplying by negative one without flipping
5. Wrong brackets at the zeros

🌱 Small wins — tick as you go

- ○ I can flip an inequality when I multiply by a negative, and factor $x^2 + bx + c$. (5′)
- ○ I can get any quadratic inequality into the form "expression compared with 0" and name its zeros. (6′)
- ○ I can follow, then finish, a zeros–sketch–pick solution. (8′)
- ○ I can solve one on my own with at most two hints and write it in interval notation. (8′)
- ○ I can spot the five classic mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Solve $-3x + 6 > 12$.
A. $x > -2$ B. $x < -2$ C. $x > 2$ D. $x < 6$

If you missed it — **Inequalities: when the sign flips**: Solve like an equation, EXCEPT: multiplying or dividing both sides by a negative number flips the sign. $-2x > 6 \rightarrow x < -3$. Test a number to be sure: $x = -4$ gives $8 > 6$ ✓.

1. $5 - 3x \leq 11 \rightarrow -3x \leq 6 \rightarrow x \geq -2$ (divided by -3: flip).
2. Number line: closed dot for $\leq, \geq$; open dot for $<, >$.

2. Factor $x^2 - x - 20$.
A. $(x-4)(x+5)$ B. $(x-10)(x+2)$ C. $(x+4)(x-5)$ D. $(x-4)(x-5)$

If you missed it — **Factoring $x^2 + bx + c$ (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12 \rightarrow -4$ and $3 \rightarrow (x-4)(x+3)$. Check by expanding.

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7 \rightarrow (x+2)(x+5)$.
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.

3. Is $(x-3)(x+1)$ positive or negative when $x = 0$?
A. Negative (-3) B. Positive (3) C. Zero D. Positive (4)

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

4. Where does the graph of $y = (x+2)(x-6)$ cross the x-axis?
A. $x = 2$ and $x = -6$ B. $x = -12$ only C. $x = -2$ and $x = 6$ D. $y = -12$

If you missed it — **Reading a graph: points, intercepts, slope**: A point is (x, y): across first, then up. x-intercepts are where $y = 0$ (the graph crosses the horizontal axis); the y-intercept is where $x = 0$. Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

1. For $y = x^2 - 4$: y-intercept at $x = 0$ is -4; x-intercepts solve $x^2 - 4 = 0$: $x = \pm 2$.
2. Above the x-axis means $y > 0$; below means $y < 0$.

Answers: 1: B · 2: C · 3: A · 4: C

🔍 Common-errors sheet

1. Picking the wrong region

Solve $x^2 - 4x - 21 > 0$.

1 $(x - 7)(x + 3) > 0$

2 Zeros -3 and 7

3 So $-3 < x < 7$

✍ Which line goes wrong first, and why?

2. Square-rooting an inequality

Solve $x^2 \geq 25$.

1 Take square roots: $x \geq \pm 5$

2 So $x \geq 5$ or $x \geq -5$

✍ Which line goes wrong first, and why?

3. Factoring before comparing with zero

Solve $x^2 + 2x < 24$.

1 $x(x + 2) < 24$

2 So $x < 24$ or $x + 2 < 24$

3 $x < 24$

✍ Which line goes wrong first, and why?

4. Multiplying by negative one without flipping

Solve $-x^2 + x + 12 > 0$.

1 Multiply by -1: $x^2 - x - 12 > 0$

2 $(x - 4)(x + 3) > 0$

3 $x < -3$ or $x > 4$

✍ Which line goes wrong first, and why?

5. Wrong brackets at the zeros

Solve $x^2 - 2x - 8 \geq 0$.

1 $(x - 4)(x + 2) \geq 0$, zeros -2 and 4

2 Opens up, so outside

3 $(-\infty, -2) \cup (4, \infty)$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 3 —** The answer "between the zeros" feels natural because the zeros are what you just found. But an upward parabola is ABOVE the axis outside its zeros. Sketch or test $x = 0$: $-21 > 0$ ×, so the middle is wrong. Answer: $x < -3$ or $x > 7$.
2. **line 1 —** It copies the equation habit $x^2 = 25 \Rightarrow x = \pm 5$. With $\geq$, the -5 side runs the other way: $x = -10$ works but is not ≥ -5. $x^2 - 25 \geq 0$, $(x - 5)(x + 5) \geq 0$, opens up, outside: $x \leq -5$ or $x \geq 5$.
3. **line 2 —** The sign rules for a product only work against 0. Knowing $x(x + 2)$ is below 24 says nothing about each factor. $x^2 + 2x - 24 < 0$, $(x + 6)(x - 4) < 0$, opens up, between: $-6 < x < 4$.
4. **line 1 —** Making a positive is a good move, but it changes the direction of every inequality, the same rule as $-2x > 6 \Rightarrow x < -3$. $x^2 - x - 12 < 0$, so between the zeros: $-3 < x < 4$. (Or keep the original: it opens down and is positive between its zeros.)
5. **line 3 —** The zeros are the points where the expression equals 0, and $\geq$ asks for "0 or more", so they belong in the answer. Square brackets at both zeros: $(-\infty, -2] \cup [4, \infty)$. Check $x = 4$: $16 - 8 - 8 = 0 \geq 0$ ✓.