Linear inequalities & interval notation

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Where are the x terms? Are there brackets or fractions? Is the sign strict ($<$, $>$) or does it include the endpoint ($\leq$, $\geq$)?
2. 🎯 **Asked** Every x that makes it true: a whole set of numbers, written in interval notation, set notation or on a number line.
3. ✏️ **Picture it** A number line with the boundary number, an open or closed dot, and shading on one side.
4. 🧰 **Tool** Solve it like an equation. Flip the sign ONLY when you multiply or divide by a negative, or move x to the side where its coefficient is positive.
5. ✅ **Check** Test one number inside your answer and one outside in the ORIGINAL. For $\leq$ or $\geq$, the boundary must work too.

⭐ Must know

- Solve like an equation; multiply or divide by a NEGATIVE → flip the sign.
- $\leq$, $\geq$: closed dot, square bracket. $<$, $>$: open dot, round bracket. ∞ always round.
- Double inequality: do the same step to all three parts.

$$a < b \text{ and } k < 0 \;\Rightarrow\; ka > kb$$

$$x \leq 4 \;\Leftrightarrow\; (-\infty, 4]$$

🔍 Watch out for

1. Not flipping when dividing by a negative
2. Flipping because the number is negative
3. Multiplying the minus sign into only the first term
4. Using a square bracket at infinity
5. Leaving the brackets at their old ends after a flip

🌱 Small wins — tick as you go

- ○ I can solve $5x - 3 = 12$, expand $-3(x - 4)$, and say what happens to $3 < 5$ when both sides are multiplied by -2. (5′)
- ○ I can ask the five starting questions for a linear inequality. (5′)
- ○ I can follow, then finish, a solve–flip–graph–test solution. (8′)
- ○ I can solve an inequality with fractions on my own with at most two hints. (8′)
- ○ I can spot the five classic inequality mistakes. (8′)
- ○ I can do the Level 1–2 practice questions and write each answer in interval notation. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Solve $5x - 3 = 12$.
A. $x = 3$ B. $x = \frac{9}{5}$ C. $x = 15$ D. $x = -3$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x - 2) = x + 8 \rightarrow 3x - 6 = x + 8 \rightarrow 2x = 14 \rightarrow x = 7$.
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30 \rightarrow x = 6$.
3. Check by substituting: $3(7 - 2) = 15$ and $7 + 8 = 15$ ✓.

2. Which statement is true?
A. $-6 > -2$ B. $-6 < -2$ C. $6 < 2$ D. $-6 = -2$

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

3. Start with $3 < 5$ and multiply both sides by -2. Which statement is true?
A. $-6 < -10$ B. $-6 > -10$ C. $6 > 10$ D. $-6 = -10$

If you missed it — **Inequalities: when the sign flips**: Solve like an equation, EXCEPT: multiplying or dividing both sides by a negative number flips the sign. $-2x > 6 \rightarrow x < -3$. Test a number to be sure: $x = -4$ gives $8 > 6$ ✓.

1. $5 - 3x \leq 11 \rightarrow -3x \leq 6 \rightarrow x \geq -2$ (divided by -3: flip).
2. Number line: closed dot for $\leq, \geq$; open dot for $<, >$.

4. Expand $-3(x - 4)$.
A. $-3x - 12$ B. $-3x + 12$ C. $-3x + 4$ D. $3x - 12$

If you missed it — **Distributing (and the minus sign)**: Multiply EVERY term inside the bracket: $3(x - 4) = 3x - 12$. A minus in front changes every sign inside: $-(x - 5) = -x + 5$, and $2 - (x + 3) = 2 - x - 3 = -x - 1$.

1. $-2(3x - 1) = -6x + 2$ (the -1 becomes $+2$).
2. $x(x + 2) - 3(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$.

5. What is the lowest common denominator of $\frac{x}{4}$ and $\frac{x}{6}$?
A. 24 B. 12 C. 10 D. 2

If you missed it — **Lowest common denominator**: Factor each denominator; the LCD uses every factor the HIGHEST number of times it appears. For 6 and 4: $2 \cdot 3$ and $2^2 \rightarrow 2^2 \cdot 3 = 12$. For x and x^2: x^2. For $(x + 1)$ and $(x - 2)$: $(x + 1)(x - 2)$.

1. $\frac{1}{2x}$ and $\frac{1}{3x^2}$: LCD $= 6x^2$.
2. Multiply top AND bottom by what is missing: $\frac{1}{2x} = \frac{3x}{6x^2}$.

Answers: 1: A · 2: B · 3: B · 4: B · 5: B

🔍 Common-errors sheet

1. Not flipping when dividing by a negative

Solve $5 - 4x > 17$.

1 $-4x > 12$

2 $x > -3$

✍ Which line goes wrong first, and why?

2. Flipping because the number is negative

Solve $3x + 5 < -7$.

1 $3x < -12$

2 $x > -4$

✍ Which line goes wrong first, and why?

3. Multiplying the minus sign into only the first term

Solve $10 - 2(x - 3) \leq 4$.

1 $10 - 2x - 6 \leq 4$

2 $-2x \leq 0$

3 $x \geq 0$

✍ Which line goes wrong first, and why?

4. Using a square bracket at infinity

Solve $2x - 1 \geq 11$ and write the answer in interval notation.

1 $2x \geq 12$

2 $x \geq 6$

3 $[6, \infty]$

✍ Which line goes wrong first, and why?

5. Leaving the brackets at their old ends after a flip

Solve $-7 \leq 1 - 2x < 5$ and write the answer in interval notation.

1 $-8 \leq -2x < 4$

2 Divide by -2 and flip: $4 \geq x > -2$

3 Interval: $[-2, 4)$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** It was treated exactly like an equation. Dividing by -4 swaps the order of numbers, so the sign must turn around. $x < -3$. Test $x = -4$: $5 + 16 = 21 > 17$ ✓.
2. **line 2 —** The -12 looked like a reason to flip. What matters is the number you divide BY: here it is $+3$, so the sign stays. $x < -4$. Test $x = -5$: $-15 + 5 = -10 < -7$ ✓.
3. **line 1 —** The -2 multiplies BOTH terms in the bracket: $-2 \times (-3) = +6$. $10 - 2x + 6 \leq 4$, $-2x \leq -12$, so $x \geq 6$.
4. **line 3 —** Both ends got the same bracket. Infinity is not a number you can reach, so it can never be included. $[6, \infty)$: square at 6 (included), round at ∞.
5. **line 3 —** The square bracket stayed on the left, where the $\leq$ used to be. After the flip, the "or equal" belongs to 4, not to -2. Rewrite from small to large: $-2 < x \leq 4$, so $(-2, 4]$.