Linear inequalities & interval notation

一次不等式与区间记号

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Where are the x terms? Are there brackets or fractions? Is the sign strict ($<$, $>$) or does it include the endpoint ($\leq$, $\geq$)?
   > x 项在哪边？有没有括号或分数？不等号是严格的（$<$、$>$），还是包含端点（$\leq$、$\geq$）？
2. 🎯 **Asked · 求什么** Every x that makes it true: a whole set of numbers, written in interval notation, set notation or on a number line.
   > 让它成立的所有 x：是一整组数，用区间记号、集合记号或数轴表示。
3. ✏️ **Picture it · 画出来** A number line with the boundary number, an open or closed dot, and shading on one side.
   > 一条数轴：标出分界点，画空心点或实心点，再涂一侧。
4. 🧰 **Tool · 用什么工具** Solve it like an equation. Flip the sign ONLY when you multiply or divide by a negative, or move x to the side where its coefficient is positive.
   > 像解方程一样解。「只有」乘或除以负数时才变号；也可以把 x 移到系数为正的一边。
5. ✅ **Check · 检验** Test one number inside your answer and one outside in the ORIGINAL. For $\leq$ or $\geq$, the boundary must work too.
   > 在答案里外各取一个数，代回「原」不等式。$\leq$ 或 $\geq$ 时，分界点本身也要成立。

⭐ Must know

必须记住

- Solve like an equation; multiply or divide by a NEGATIVE → flip the sign.
  > 像解方程一样解；乘或除以负数 → 不等号反向。
- $\leq$, $\geq$: closed dot, square bracket. $<$, $>$: open dot, round bracket. ∞ always round.
  > $\leq$, $\geq$：实心点、方括号。$<$, $>$：空心点、圆括号。∞ 永远用圆括号。
- Double inequality: do the same step to all three parts.
  > 连写不等式：每一步对三部分同时做。

$$a < b \text{ and } k < 0 \;\Rightarrow\; ka > kb$$

$$x \leq 4 \;\Leftrightarrow\; (-\infty,\, 4]$$

🔍 Watch out for

当心

1. Not flipping when dividing by a negative
   > 除以负数时没有变号
2. Flipping because the number is negative
   > 因为数是负的就变号
3. Multiplying the minus sign into only the first term
   > 负号只乘了括号里的第一项
4. Using a square bracket at infinity
   > 无穷大用了方括号
5. Leaving the brackets at their old ends after a flip
   > 变号后括号还留在原来的一端

🌱 Small wins — tick as you go

一小步一小步——做完打勾

○ I can solve $5x - 3 = 12$, expand $-3(x - 4)$, and say what happens to $3 < 5$ when both sides are multiplied by -2.

> 我会解 $5x - 3 = 12$，会展开 $-3(x - 4)$，也知道 $3 < 5$ 两边乘 -2 后会怎样。

(5′)

○ I can ask the five starting questions for a linear inequality.

> 我能对一元一次不等式问出五个入手问题。

(5′)

○ I can follow, then finish, a solve–flip–graph–test solution.

> 我能看懂并补全「解—变号—画数轴—代数检验」的解法。

(8′)

○ I can solve an inequality with fractions on my own with at most two hints.

> 我能在最多两个提示下独立解一道含分数的不等式。

(8′)

○ I can spot the five classic inequality mistakes.

> 我能找出不等式的五个经典错误。

(8′)

○ I can do the Level 1–2 practice questions and write each answer in interval notation.

> 我能做出 1–2 级练习题，并用区间记号写出每个答案。

(10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Solve $5x - 3 = 12$.

> 解 $5x - 3 = 12$。

A. $x = 3$ B. $x = \frac{9}{5}$ C. $x = 15$ D. $x = -3$

If you missed it · 如果做错了 — **Solving a linear equation**

> **解一元一次方程**

: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

> 两边做同样的事。先去括号，把含 x 的项移到一边、数字移到另一边，最后除以系数。有分数时先把每一项都乘以公分母。

1. $3(x - 2) = x + 8 \to 3x - 6 = x + 8 \to 2x = 14 \to x = 7$.
   > $3(x - 2) = x + 8 \to 3x - 6 = x + 8 \to 2x = 14 \to x = 7$。
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30 \to x = 6$.
   > $\frac{x}{2} + \frac{x}{3} = 5$：每项乘 6 → $3x + 2x = 30 \to x = 6$。
3. Check by substituting: $3(7 - 2) = 15$ and $7 + 8 = 15$ ✓.
   > 代回检验：$3(7 - 2) = 15$，$7 + 8 = 15$ ✓。

2. Which statement is true?

> 下面哪个式子成立?

A. $-6 > -2$ B. $-6 < -2$ C. $6 < 2$ D. $-6 = -2$

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

> 减去负数等于加上它：$5 - (-3) = 5 + 3 = 8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2 = 16$，但 $-4^2 = -16$（先算平方）。

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
   > $-7 - (-2)$：把「减负」改成「加」：$-7 + 2 = -5$。
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
   > $(-3)(-5) = 15$（同号），$(-3)(5) = -15$（异号）。
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.
   > 把 $x = -3$ 代入 x^2：先加括号，$(-3)^2 = 9$。

3. Start with $3 < 5$ and multiply both sides by -2. Which statement is true?

从 $3 < 5$ 出发，两边都乘 -2。哪个式子成立?

A. $-6 < -10$ B. $-6 > -10$ C. $6 > 10$ D. $-6 = -10$

If you missed it · 如果做错了 — **Inequalities: when the sign flips**

不等式：什么时候变号

: Solve like an equation, EXCEPT: multiplying or dividing both sides by a negative number flips the sign. $-2x > 6 \rightarrow x < -3$. Test a number to be sure: $x = -4$ gives $8 > 6$ ✓.

像解方程一样解，唯一例外：两边同乘或同除以负数，不等号要反向。$-2x > 6 \rightarrow x < -3$。代一个数确认：$x = -4$ 得 $8 > 6$ ✓。

1. $5 - 3x \leq 11 \rightarrow -3x \leq 6 \rightarrow x \geq -2$ (divided by -3: flip).
   $5 - 3x \leq 11 \rightarrow -3x \leq 6 \rightarrow x \geq -2$（除以 -3：变号）。
2. Number line: closed dot for $\leq, \geq$; open dot for $<, >$.
   数轴：$\leq, \geq$ 用实心点；$<, >$ 用空心点。

4. Expand $-3(x - 4)$.

展开 $-3(x - 4)$。

A. $-3x - 12$ B. $-3x + 12$ C. $-3x + 4$ D. $3x - 12$

If you missed it · 如果做错了 — **Distributing (and the minus sign)**

去括号（尤其是负号）

: Multiply EVERY term inside the bracket: $3(x - 4) = 3x - 12$. A minus in front changes every sign inside: $-(x - 5) = -x + 5$, and $2 - (x + 3) = 2 - x - 3 = -x - 1$.

括号里的每一项都要乘到：$3(x - 4) = 3x - 12$。前面是负号时，括号里每一项都变号：$-(x - 5) = -x + 5$，$2 - (x + 3) = 2 - x - 3 = -x - 1$。

1. $-2(3x - 1) = -6x + 2$ (the -1 becomes $+2$).
   $-2(3x - 1) = -6x + 2$（-1 变成 $+2$）。
2. $x(x + 2) - 3(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$.
   $x(x + 2) - 3(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$。

5. What is the lowest common denominator of $\frac{x}{4}$ and $\frac{x}{6}$?

$\frac{x}{4}$ 和 $\frac{x}{6}$ 的最小公分母是多少?

A. 24 B. 12 C. 10 D. 2

If you missed it · 如果做错了 — **Lowest common denominator**

最简公分母

: Factor each denominator; the LCD uses every factor the HIGHEST number of times it appears. For 6 and 4: $2 \cdot 3$ and $2^2 \rightarrow 2^2 \cdot 3 = 12$. For x and x^2: x^2. For $(x + 1)$ and $(x - 2)$: $(x + 1)(x - 2)$.

把每个分母分解；最简公分母取每个因式出现的最高次数。6 和 4：$2 \cdot 3$ 与 $2^2 \rightarrow 2^2 \cdot 3 = 12$。$x$ 和 x^2：x^2。$(x + 1)$ 和 $(x - 2)$：$(x + 1)(x - 2)$。

1. $\frac{1}{2x}$ and $\frac{1}{3x^2}$: LCD $= 6x^2$.
   $\frac{1}{2x}$ 与 $\frac{1}{3x^2}$：最简公分母 $= 6x^2$。
2. Multiply top AND bottom by what is missing: $\frac{1}{2x} = \frac{3x}{6x^2}$.
   分子分母同乘缺的部分：$\frac{1}{2x} = \frac{3x}{6x^2}$。

Answers · 答案: 1: A · 2: B · 3: B · 4: B · 5: B

🔍 Common-errors sheet

常见错误单

1. Not flipping when dividing by a negative

> 除以负数时没有变号

Solve $5 - 4x > 17$.

> 解 $5 - 4x > 17$。

1 $-4x > 12$

2 $x > -3$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Flipping because the number is negative

> 因为数是负的就变号

Solve $3x + 5 < -7$.

> 解 $3x + 5 < -7$。

1 $3x < -12$

2 $x > -4$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Multiplying the minus sign into only the first term

> 负号只乘了括号里的第一项

Solve $10 - 2(x - 3) \leq 4$.

> 解 $10 - 2(x - 3) \leq 4$。

1 $10 - 2x - 6 \leq 4$

2 $-2x \leq 0$

3 $x \geq 0$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Using a square bracket at infinity

> **无穷大用了方括号**

Solve $2x - 1 \geq 11$ and write the answer in interval notation.

> 解 $2x - 1 \geq 11$，用区间记号写出答案。

1 $2x \geq 12$

2 $x \geq 6$

3 $[6, \infty]$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Leaving the brackets at their old ends after a flip

> **变号后括号还留在原来的一端**

Solve $-7 \leq 1 - 2x < 5$ and write the answer in interval notation.

> 解 $-7 \leq 1 - 2x < 5$，用区间记号写出答案。

1 $-8 \leq -2x < 4$

2 Divide by -2 and flip: $4 \geq x > -2$

> 除以 -2 并变号：$4 \geq x > -2$

3 Interval: $[-2, 4)$

> 区间：$[-2, 4)$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 2 —** It was treated exactly like an equation. Dividing by -4 swaps the order of numbers, so the sign must turn around.
   > 完全当成方程来解了。除以 -4 会把数的大小顺序倒过来，所以不等号要反向。

   $x < -3$. Test $x = -4$: $5 + 16 = 21 > 17$ ✓.
   > $x < -3$。检验 $x = -4$：$5 + 16 = 21 > 17$ ✓。
2. **line · 第 2 —** The -12 looked like a reason to flip. What matters is the number you divide BY: here it is $+3$, so the sign stays.
   > 看到 -12 就以为要变号。关键是你「除以」的那个数：这里是 $+3$，不等号不变。

   $x < -4$. Test $x = -5$: $-15 + 5 = -10 < -7$ ✓.
   > $x < -4$。检验 $x = -5$：$-15 + 5 = -10 < -7$ ✓。
3. **line · 第 1 —** The -2 multiplies BOTH terms in the bracket: $-2 \times (-3) = +6$.
   > -2 要乘括号里的「两项」：$-2 \times (-3) = +6$。

   $10 - 2x + 6 \le 4$, $-2x \le -12$, so $x \ge 6$.
   > $10 - 2x + 6 \le 4$，$-2x \le -12$，所以 $x \ge 6$。
4. **line · 第 3 —** Both ends got the same bracket. Infinity is not a number you can reach, so it can never be included.
   > 两端用了一样的括号。无穷大不是能到达的数，永远不能被包含。

   $[6, \infty)$: square at 6 (included), round at ∞.
   > $[6, \infty)$：6 处用方括号（包含），∞ 处用圆括号。
5. **line · 第 3 —** The square bracket stayed on the left, where the $\le$ used to be. After the flip, the "or equal" belongs to 4, not to -2.
   > 方括号还留在左边，也就是原来 $\le$ 的位置。变号以后，「等于」跟着的是 4，不是 -2。

   Rewrite from small to large: $-2 < x \le 4$, so $(-2, 4]$.
   > 按从小到大重写：$-2 < x \le 4$，即 $(-2, 4]$。