Domain & range limits in context

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** What does the input stand for (time, a width, a number of price increases)? Write its limits first: time starts at 0 and stops when the object lands, every length is more than 0, counts are whole numbers.
2. 🎯 **Asked** The domain, the range, or a window ("at least", "higher than")? Answer as an inequality with units.
3. ✏️ **Picture it** Sketch only the part of the arch that is real: the start point, the vertex and the end point. For a window, draw the horizontal line $y =$ target.
4. 🧰 **Tool** Domain: solve "$h = 0$" or "each length > 0". Range: the vertex value and the values at the ends of the domain. Window: move everything to one side, factor, use a sign chart (flip the sign if you divide by a negative).
5. ✅ **Check** Every answer must sit inside the domain. Test one value from your answer in the original model.

⭐ Must know

- Domain in context: time from 0 to landing, lengths > 0, counts are whole numbers.
- Range in context: the vertex value (if inside the domain) and the endpoint values.
- "At least / more than" → solve the inequality, then cut it to the domain.

$$h(t) \geq 0 \text{ and } t \geq 0$$

🔍 Watch out for

1. Writing all real numbers for a word problem
2. Keeping the negative time
3. Taking the range from the endpoints only
4. Using the starting height as the lowest value
5. Not flipping the sign when dividing by a negative

🌱 Small wins — tick as you go

- ○ I can flip an inequality when I divide by a negative, solve $t^2 - 5t + 4 = 0$, and read the top point of a downward parabola. (5′)
- ○ I can write the real-life limits on the input (time, lengths, counts) before doing any algebra. (5′)
- ○ I can follow, then finish, a domain-and-range solution for a thrown ball. (8′)
- ○ I can find the domain and range of a launch from a cliff with at most two hints. (8′)
- ○ I can spot the five classic domain-and-range mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions, giving every answer with units and a reason. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Solve $-2x > 8$.
A. $x > -4$ B. $x < -4$ C. $x > 4$ D. $x < 4$

If you missed it — **Inequalities: when the sign flips**: Solve like an equation, EXCEPT: multiplying or dividing both sides by a negative number flips the sign. $-2x > 6 \rightarrow x < -3$. Test a number to be sure: $x = -4$ gives $8 > 6$ ✓.

1. $5 - 3x \leq 11 \rightarrow -3x \leq 6 \rightarrow x \geq -2$ (divided by -3: flip).
2. Number line: closed dot for $\leq, \geq$; open dot for $<, >$.

2. Solve $t^2 - 5t + 4 = 0$.
A. $t = 1$ or $t = 4$ B. $t = -1$ or $t = -4$ C. $t = 2$ only D. $t = 5$ or $t = 4$

If you missed it — **Factoring $x^2 + bx + c$ (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12 \rightarrow -4$ and $3 \rightarrow (x - 4)(x + 3)$. Check by expanding.

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7 \rightarrow (x + 2)(x + 5)$.
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.

3. The graph of $y = -2(x - 3)^2 + 18$ opens down. What is its highest point?
A. $(3, 18)$ B. $(-3, 18)$ C. $(18, 3)$ D. $(3, -18)$

If you missed it — **Reading a graph: points, intercepts, slope**: A point is (x, y): across first, then up. x-intercepts are where $y = 0$ (the graph crosses the horizontal axis); the y-intercept is where $x = 0$. Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

1. For $y = x^2 - 4$: y-intercept at $x = 0$ is -4; x-intercepts solve $x^2 - 4 = 0$: $x = \pm 2$.
2. Above the x-axis means $y > 0$; below means $y < 0$.

4. Evaluate $-5t^2 + 20t$ when $t = -1$.
A. -25 B. 15 C. 25 D. -15

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

Answers: 1: B · 2: A · 3: A · 4: A

🔍 Common-errors sheet

1. Writing all real numbers for a word problem

A rectangle has perimeter 20 m and width x m, so $A = x(10 - x)$. State the domain and range.

1 $A = x(10 - x)$ is a parabola that opens down.

2 Domain: all real numbers.

3 Range: $A \leq 25$.

✍ Which line goes wrong first, and why?

2. Keeping the negative time

A ball's height is $h(t) = -5t^2 + 15t + 20$. State the domain.

1 $-5(t^2 - 3t - 4) = 0$

2 $(t - 4)(t + 1) = 0$, so $t = 4$ or $t = -1$

3 Domain: $-1 \leq t \leq 4$

✍ Which line goes wrong first, and why?

3. Taking the range from the endpoints only

$h(t) = -5t^2 + 20t + 25$ for $0 \leq t \leq 5$. Find the range.

1 $h(0) = 25$ and $h(5) = 0$

2 Range: $0 \leq h \leq 25$

✍ Which line goes wrong first, and why?

4. Using the starting height as the lowest value

A ball is thrown from a 40 m roof: $h(t) = -5t^2 + 10t + 40$. Find the range.

1 Vertex: $t = 1$, $h = 45$

2 The lowest value is the start, $h(0) = 40$

3 Range: $40 \leq h \leq 45$

✍ Which line goes wrong first, and why?

5. Not flipping the sign when dividing by a negative

$h(t) = -5t^2 + 30t$. When is the ball higher than 25 m?

1 $-5t^2 + 30t - 25 > 0$

2 Divide by -5: $t^2 - 6t + 5 > 0$

3 $(t - 1)(t - 5) > 0$: $t < 1$ or $t > 5$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** The formula works for any x, but a rectangle does not: a width of -3 or 12 m makes a side negative. Width $x > 0$ and length $10 - x > 0$: domain $0 < x < 10$. Vertex $x = 5$ gives $A = 25$, so the range is $0 < A \leq 25$ m².
2. **line 3 —** The algebra is right, but the ball does not exist before it is thrown. $t = -1$ is a zero of the formula, not a moment in the story. Reject $t = -1$. Domain: $0 \leq t \leq 4$ s.
3. **line 2 —** An arch is highest in the middle, not at an end. The vertex was never checked. Vertex $t = 2$: $h(2) = -20 + 40 + 25 = 45$. Range: $0 \leq h \leq 45$.
4. **line 2 —** The ball does not stop at roof height. It falls past the roof all the way to the ground. $h = 0$ when $t^2 - 2t - 8 = 0$, so $t = 4$. The lowest height is 0, so the range is $0 \leq h \leq 45$ m.
5. **line 2 —** Dividing both sides by a negative number reverses the inequality. The answer "$t < 1$ or $t > 5$" also says the ball is highest at the start and the end, which is backwards. $t^2 - 6t + 5 < 0$, so $1 < t < 5$. This is inside the domain $0 \leq t \leq 6$: above 25 m for 4 s.