Domain & range limits in context

实际问题中的定义域与值域

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** What does the input stand for (time, a width, a number of price increases)? Write its limits first: time starts at 0 and stops when the object lands, every length is more than 0, counts are whole numbers.
   > 输入量代表什么（时间、宽度、涨价次数）？先写出它的限制：时间从 0 开始、落地就停；每条长度都大于 0；个数只能是整数。
2. 🎯 **Asked · 求什么** The domain, the range, or a window ("at least", "higher than")? Answer as an inequality with units.
   > 求定义域、值域，还是一个时间段（「至少」「高于」）？用带单位的不等式作答。
3. ✏️ **Picture it · 画出来** Sketch only the part of the arch that is real: the start point, the vertex and the end point. For a window, draw the horizontal line $y =$ target.
   > 只画拱形中真实存在的那一段：起点、顶点、终点。求时间段时，再画一条水平线 $y =$ 目标值。
4. 🧰 **Tool · 用什么工具** Domain: solve "$h = 0$" or "each length > 0". Range: the vertex value and the values at the ends of the domain. Window: move everything to one side, factor, use a sign chart (flip the sign if you divide by a negative).
   > 定义域：解「$h = 0$」或「每条长度 > 0」。值域：顶点值和定义域两端的值。时间段：移到一边、分解、列符号表（除以负数要变号）。
5. ✅ **Check · 检验** Every answer must sit inside the domain. Test one value from your answer in the original model.
   > 每个答案都要落在定义域里。从答案里取一个值，代回原模型检验。

⭐ Must know

必须记住

- Domain in context: time from 0 to landing, lengths > 0, counts are whole numbers.
  > 情境中的定义域：时间从 0 到落地，长度 > 0，个数取整数。
- Range in context: the vertex value (if inside the domain) and the endpoint values.
  > 情境中的值域：顶点值（若在定义域内）和端点值。
- "At least / more than" → solve the inequality, then cut it to the domain.
  > 「至少 / 超过」→ 解不等式，再截到定义域内。

$$h(t) \geq 0 \text{ and } t \geq 0$$

🔍 Watch out for

当心

1. Writing all real numbers for a word problem
   > 应用题定义域写成全体实数
2. Keeping the negative time
   > 留下了负的时间
3. Taking the range from the endpoints only
   > 只用端点求值域
4. Using the starting height as the lowest value
   > 把起始高度当最小值
5. Not flipping the sign when dividing by a negative
   > 除以负数时没变号

🌱 Small wins — tick as you go

一小步一小步——做完打勾

○ I can flip an inequality when I divide by a negative, solve $t^2 - 5t + 4 = 0$, and read the top point of a downward parabola.

> 我会在除以负数时给不等式变号，会解 $t^2 - 5t + 4 = 0$，也能读出开口向下的抛物线的最高点。

(5′)

○ I can write the real-life limits on the input (time, lengths, counts) before doing any algebra.

> 我能在做代数之前，先写出输入量的实际限制（时间、长度、个数）。

(5′)

○ I can follow, then finish, a domain-and-range solution for a thrown ball.

> 我能看懂并补全一道抛球问题的定义域和值域。

(8′)

○ I can find the domain and range of a launch from a cliff with at most two hints.

> 我能在最多两个提示下，求出从悬崖抛出物体的定义域和值域。

(8′)

○ I can spot the five classic domain-and-range mistakes in someone else's work.

> 我能在别人的解答里找出定义域与值域的五个经典错误。

(8′)

○ I can do the Level 1–2 practice questions, giving every answer with units and a reason.

> 我能做出 1–2 级练习题，每个答案都带单位和理由。

(10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Solve $-2x > 8$.

> 解 $-2x > 8$。

A. $x > -4$ B. $x < -4$ C. $x > 4$ D. $x < 4$

If you missed it · 如果做错了 — **Inequalities: when the sign flips**

> **不等式：什么时候变号**

: Solve like an equation, EXCEPT: multiplying or dividing both sides by a negative number flips the sign. $-2x > 6 \to x < -3$. Test a number to be sure: $x = -4$ gives $8 > 6$ ✓.

> 像解方程一样解，唯一例外：两边同乘或同除以负数，不等号要反向。$-2x > 6 \to x < -3$。代一个数确认：$x = -4$ 得 $8 > 6$ ✓。

1. $5 - 3x \le 11 \to -3x \le 6 \to x \ge -2$ (divided by -3: flip).
   > $5 - 3x \le 11 \to -3x \le 6 \to x \ge -2$（除以 -3：变号）。
2. Number line: closed dot for $\le, \ge$; open dot for $<, >$.
   > 数轴：$\le, \ge$ 用实心点；$<, >$ 用空心点。

2. Solve $t^2 - 5t + 4 = 0$.

> 解 $t^2 - 5t + 4 = 0$。

A. $t = 1$ or $t = 4$ B. $t = -1$ or $t = -4$ C. $t = 2$ only D. $t = 5$ or $t = 4$

If you missed it · 如果做错了 — **Factoring $x^2 + bx + c$ (product and sum)**

> **分解 $x^2 + bx + c$（积与和）**

: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12 \to -4$ and $3 \to (x - 4)(x + 3)$. Check by expanding.

> 先提公因式。再找两个数，相乘等于 c、相加等于 b：$x^2 - x - 12 \to -4$ 和 $3 \to (x - 4)(x + 3)$。展开检验。

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7 \to (x + 2)(x + 5)$.
   > $x^2 + 7x + 10$：$2 \times 5 = 10$，$2 + 5 = 7 \to (x + 2)(x + 5)$。
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.
   > 若 $c < 0$，两个数异号；若 $c > 0$，两个数与 b 同号。

3. The graph of $y = -2(x - 3)^2 + 18$ opens down. What is its highest point?

> $y = -2(x - 3)^2 + 18$ 的图象开口向下。它的最高点是哪一点?

A. $(3, 18)$ B. $(-3, 18)$ C. $(18, 3)$ D. $(3, -18)$

If you missed it · 如果做错了 — **Reading a graph: points, intercepts, slope**

> **读图：点、截距、斜率**

: A point is (x, y): across first, then up. x-intercepts are where $y = 0$ (the graph crosses the horizontal axis); the y-intercept is where $x = 0$. Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

> 点写作 (x, y)：先横后纵。x 截距是 $y = 0$ 的地方（图象穿过横轴处）；y 截距是 $x = 0$ 的地方。斜率 $= \frac{\text{纵向变化}}{\text{横向变化}} = \frac{y_2 - y_1}{x_2 - x_1}$。

1. For $y = x^2 - 4$: y-intercept at $x = 0$ is -4; x-intercepts solve $x^2 - 4 = 0$: $x = \pm 2$.
   > $y = x^2 - 4$：令 $x = 0$ 得 y 截距 -4；令 $y = 0$ 解 $x^2 - 4 = 0$ 得 x 截距 $x = \pm 2$。
2. Above the x-axis means $y > 0$; below means $y < 0$.
   > 在 x 轴上方表示 $y > 0$；下方表示 $y < 0$。

4. Evaluate $-5t^2 + 20t$ when $t = -1$.

> 当 $t = -1$ 时，求 $-5t^2 + 20t$ 的值。

A. -25 B. 15 C. 25 D. -15

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

> 减去负数等于加上它：$5 - (-3) = 5 + 3 = 8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2 = 16$，但 $-4^2 = -16$（先算平方）。

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
   > $-7 - (-2)$：把「减负」改成「加」：$-7 + 2 = -5$。
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
   > $(-3)(-5) = 15$（同号），$(-3)(5) = -15$（异号）。
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.
   > 把 $x = -3$ 代入 x^2：先加括号，$(-3)^2 = 9$。

Answers · 答案: 1: B · 2: A · 3: A · 4: A

🔍 Common-errors sheet

常见错误单

1. Writing all real numbers for a word problem

应用题定义域写成全体实数

A rectangle has perimeter 20 m and width x m, so $A = x(10 - x)$. State the domain and range.

长方形周长 20 m，宽 x m，所以 $A = x(10 - x)$。写出定义域和值域。

1 $A = x(10 - x)$ is a parabola that opens down.

$A = x(10 - x)$ 是开口向下的抛物线。

2 Domain: all real numbers.

定义域：全体实数。

3 Range: $A \leq 25$.

值域：$A \leq 25$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Keeping the negative time

留下了负的时间

A ball's height is $h(t) = -5t^2 + 15t + 20$. State the domain.

球的高度 $h(t) = -5t^2 + 15t + 20$。写出定义域。

1 $-5(t^2 - 3t - 4) = 0$

2 $(t - 4)(t + 1) = 0$, so $t = 4$ or $t = -1$

$(t - 4)(t + 1) = 0$，所以 $t = 4$ 或 $t = -1$

3 Domain: $-1 \leq t \leq 4$

定义域：$-1 \leq t \leq 4$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Taking the range from the endpoints only

只用端点求值域

$h(t) = -5t^2 + 20t + 25$ for $0 \le t \le 5$. Find the range.

$h(t) = -5t^2 + 20t + 25$，$0 \le t \le 5$。求值域。

1 $h(0) = 25$ and $h(5) = 0$

$h(0) = 25$，$h(5) = 0$

2 Range: $0 \le h \le 25$

值域：$0 \le h \le 25$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Using the starting height as the lowest value

把起始高度当最小值

A ball is thrown from a 40 m roof: $h(t) = -5t^2 + 10t + 40$. Find the range.

从 40 m 高的屋顶抛出一个球：$h(t) = -5t^2 + 10t + 40$。求值域。

1 Vertex: $t = 1$, $h = 45$

顶点：$t = 1$，$h = 45$

2 The lowest value is the start, $h(0) = 40$

最低值是起点 $h(0) = 40$

3 Range: $40 \le h \le 45$

值域：$40 \le h \le 45$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Not flipping the sign when dividing by a negative

除以负数时没变号

$h(t) = -5t^2 + 30t$. When is the ball higher than 25 m?

$h(t) = -5t^2 + 30t$。球什么时候高于 25 m?

1 $-5t^2 + 30t - 25 > 0$

2 Divide by -5: $t^2 - 6t + 5 > 0$

除以 -5：$t^2 - 6t + 5 > 0$

3 $(t-1)(t-5) > 0$: $t < 1$ or $t > 5$

$(t-1)(t-5) > 0$：$t < 1$ 或 $t > 5$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 2 —** The formula works for any x, but a rectangle does not: a width of -3 or 12 m makes a side negative.

   > 公式对任何 x 都能算，可长方形不行：宽为 -3 或 12 米时，总有一条边是负的。

   Width $x > 0$ and length $10 - x > 0$: domain $0 < x < 10$. Vertex $x = 5$ gives $A = 25$, so the range is $0 < A \leq 25$ m².

   > 宽 $x > 0$，长 $10 - x > 0$：定义域 $0 < x < 10$。顶点 $x = 5$ 时 $A = 25$，值域 $0 < A \leq 25$ m²。

2. **line · 第 3 —** The algebra is right, but the ball does not exist before it is thrown. $t = -1$ is a zero of the formula, not a moment in the story.

   > 代数没错，但球在抛出之前并不存在。$t = -1$ 只是公式的零点，不是故事里的时刻。

   Reject $t = -1$. Domain: $0 \leq t \leq 4$ s.

   > 舍去 $t = -1$。定义域：$0 \leq t \leq 4$ 秒。

3. **line · 第 2 —** An arch is highest in the middle, not at an end. The vertex was never checked.

   > 拱形最高点在中间，不在两端。顶点根本没算。

   Vertex $t = 2$: $h(2) = -20 + 40 + 25 = 45$. Range: $0 \leq h \leq 45$.

   > 顶点 $t = 2$：$h(2) = -20 + 40 + 25 = 45$。值域：$0 \leq h \leq 45$。

4. **line · 第 2 —** The ball does not stop at roof height. It falls past the roof all the way to the ground.

   > 球不会停在屋顶高度，它会越过屋顶一直落到地面。

   $h = 0$ when $t^2 - 2t - 8 = 0$, so $t = 4$. The lowest height is 0, so the range is $0 \leq h \leq 45$ m.

   > $t^2 - 2t - 8 = 0$ 时 $h = 0$，即 $t = 4$。最低高度是 0，所以值域是 $0 \leq h \leq 45$ 米。

5. **line · 第 2 —** Dividing both sides by a negative number reverses the inequality. The answer "$t < 1$ or $t > 5$" also says the ball is highest at the start and the end, which is backwards.

   > 两边同除以负数，不等号要反向。「$t < 1$ 或 $t > 5$」等于说球在开头和结尾最高，正好反了。

   $t^2 - 6t + 5 < 0$, so $1 < t < 5$. This is inside the domain $0 \leq t \leq 6$: above 25 m for 4 s.

   > $t^2 - 6t + 5 < 0$，所以 $1 < t < 5$。它在定义域 $0 \leq t \leq 6$ 内：高于 25 m 的时间共 4 秒。