The quadratic formula

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Get 0 on one side first. Then write $a =$, $b =$, $c =$ on their own line, signs included.
2. 🎯 **Asked** Exact answers (simplified radicals) or decimals? How many decimal places?
3. ✏️ **Picture it** The answers are the x-intercepts of $y = ax^2 + bx + c$. $D = b^2 - 4ac$ says how many there are: two, one or none.
4. 🧰 **Tool** Work out $D = b^2 - 4ac$ first, with b in brackets. Then $x = \dfrac{-b \pm \sqrt{D}}{2a}$.
5. ✅ **Check** Simplify the radical, then divide EVERY term on top by the bottom. The two answers should add to $-\dfrac{b}{a}$.

⭐ Must know

- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, only after the equation is $ax^2 + bx + c = 0$.
- Read a, b, c with their signs; b^2 is never negative.
- Simplify the radical, then divide EVERY term on top by $2a$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

🔍 Watch out for

1. Dividing only part of the top
2. Reading c before moving every term
3. Getting the sign of 4ac wrong when c is negative
4. Starting with b instead of negative b
5. Dividing by a instead of 2a

🌱 Small wins — tick as you go

- ○ I can work out $b^2 - 4ac$ with negative numbers, simplify $\sqrt{28}$, and divide every term of $\frac{6+4\sqrt{2}}{2}$. (5′)
- ○ I can ask the five starting questions before I use the quadratic formula. (5′)
- ○ I can follow, then finish, a read–D–formula–simplify solution. (8′)
- ○ I can solve one on my own, exact and to the nearest hundredth, with at most two hints. (8′)
- ○ I can spot the five classic quadratic-formula mistakes. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Work out $b^2 - 4ac$ when $a = 2$, $b = -3$ and $c = -5$.
A. -31 B. 49 C. 31 D. -49

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

2. Simplify $\sqrt{28}$.
A. $2\sqrt{7}$ B. $4\sqrt{7}$ C. $7\sqrt{2}$ D. 14

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.

3. Simplify $\dfrac{6 + 4\sqrt{2}}{2}$.
A. $3 + 4\sqrt{2}$ B. $6 + 2\sqrt{2}$ C. $3 + 2\sqrt{2}$ D. $5\sqrt{2}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.

4. Write $2x^2 = 5x - 1$ in the form $ax^2 + bx + c = 0$.
A. $2x^2 + 5x - 1 = 0$ B. $2x^2 - 5x + 1 = 0$ C. $2x^2 - 5x - 1 = 0$ D. $-2x^2 - 5x + 1 = 0$

If you missed it — **Rearranging a formula**: Treat the other letters as numbers and undo operations in reverse order. $A = P(1 + r)$ for r: divide by P, then subtract 1: $r = \frac{A}{P} - 1$.

1. $y = a(x - p)^2 + q$ for a: subtract q, divide by $(x - p)^2$: $a = \frac{y - q}{(x-p)^2}$.
2. Substitute numbers first if the question gives them — fewer letters, fewer slips.

Answers: 1: B · 2: A · 3: C · 4: B

🔍 Common-errors sheet

1. Dividing only part of the top

Solve $x^2 - 8x + 4 = 0$.

1 $D = 64 - 16 = 48$

2 $x = \dfrac{8 \pm \sqrt{48}}{2} = \dfrac{8 \pm 4\sqrt{3}}{2}$

3 $x = 8 \pm 2\sqrt{3}$

✍ Which line goes wrong first, and why?

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2. Reading c before moving every term

Solve $x^2 = 2x + 5$.

1 $a = 1, b = 2, c = 5$

2 $D = 4 - 20 = -16$

3 No real solution.

✍ Which line goes wrong first, and why?

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3. Getting the sign of 4ac wrong when c is negative

Solve $x^2 + 2x - 6 = 0$.

1 $D = 4 - 4(1)(6) = 4 - 24 = -20$

2 No real solution.

✍ Which line goes wrong first, and why?

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4. Starting with b instead of negative b

Solve $x^2 - 5x + 3 = 0$.

1 $D = 25 - 12 = 13$

2 $x = \dfrac{-5 \pm \sqrt{13}}{2}$

✍ Which line goes wrong first, and why?

5. Dividing by a instead of 2a

Solve $2x^2 - 6x + 1 = 0$.

1 $D = 36 - 8 = 28$

2 $x = \dfrac{6 \pm \sqrt{28}}{2}$

3 $x = 3 \pm \sqrt{7}$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 3 —** Only the radical part looked divisible, so only it was divided. The fraction bar covers the whole top. Divide both terms by 2: $x = 4 \pm 2\sqrt{3}$.
2. **line 1 —** The numbers were read off where they sit. The formula only works for $ax^2 + bx + c = 0$, so both terms must move left first, changing sign. $x^2 - 2x - 5 = 0$, $D = 4 + 20 = 24$, $x = \dfrac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}$.
3. **line 1 —** The minus sign of c was left out. $-4(1)(-6)$ is $+24$, so D gets bigger, not smaller. $D = 4 + 24 = 28$, $x = \dfrac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$.
4. **line 2 —** "$-b$" was read as "write a minus and then 5". Here $b = -5$, so $-b = +5$. $x = \dfrac{5 \pm \sqrt{13}}{2}$. Check: the answers add to $5 = -\frac{b}{a}$.
5. **line 2 —** With $a = 2$ the bottom looked like "2" already. But the bottom is $2a = 2(2) = 4$. $x = \dfrac{6 \pm 2\sqrt{7}}{4} = \dfrac{3 \pm \sqrt{7}}{2}$.