The quadratic formula

求根公式

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Get 0 on one side first. Then write $a =$, $b =$, $c =$ on their own line, signs included.
   > 先让一边为 0，再单独写一行 $a =$、$b =$、$c =$，连同符号。
2. 🎯 **Asked · 求什么** Exact answers (simplified radicals) or decimals? How many decimal places?
   > 要精确解（化简后的根式）还是小数？保留几位？
3. ✏️ **Picture it · 画出来** The answers are the x-intercepts of $y = ax^2 + bx + c$. $D = b^2 - 4ac$ says how many there are: two, one or none.
   > 答案就是 $y = ax^2 + bx + c$ 的 x 截距。$D = b^2 - 4ac$ 告诉你有几个：两个、一个或没有。
4. 🧰 **Tool · 用什么工具** Work out $D = b^2 - 4ac$ first, with b in brackets. Then $x = \dfrac{-b \pm \sqrt{D}}{2a}$.
   > 先算 $D = b^2 - 4ac$，b 要加括号。再用 $x = \dfrac{-b \pm \sqrt{D}}{2a}$。
5. ✅ **Check · 检验** Simplify the radical, then divide EVERY term on top by the bottom. The two answers should add to $-\dfrac{b}{a}$.
   > 先化简根式，再把分子的每一项都除以分母。两个答案之和应等于 $-\dfrac{b}{a}$。

⭐ Must know

必须记住

- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, only after the equation is $ax^2 + bx + c = 0$.
  > $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$，先化成 $ax^2 + bx + c = 0$ 再用。
- Read a, b, c with their signs; b^2 is never negative.
  > a, b, c 连符号一起读；b^2 永远不是负数。
- Simplify the radical, then divide EVERY term on top by $2a$.
  > 先化简根式，再把分子每一项都除以 $2a$。

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

🔍 Watch out for

当心

1. Dividing only part of the top
   > 分子只除了一部分
2. Reading c before moving every term
   > 还没移项就读 c
3. Getting the sign of 4ac wrong when c is negative
   > c 为负时 4ac 的符号弄错
4. Starting with b instead of negative b
   > 分子开头用了 b 而不是 −b
5. Dividing by a instead of 2a
   > 分母用了 a 而不是 2a

🌱 Small wins — tick as you go

一小步一小步——做完打勾

- ○ I can work out $b^2 - 4ac$ with negative numbers, simplify $\sqrt{28}$, and divide every term of $\frac{6+4\sqrt{2}}{2}$.

  > 我会算带负数的 $b^2 - 4ac$，会化简 $\sqrt{28}$，也会把 $\frac{6+4\sqrt{2}}{2}$ 的每一项都除掉。

  (5′)

- ○ I can ask the five starting questions before I use the quadratic formula.

  > 用求根公式之前，我能先问出五个入手问题。

  (5′)

- ○ I can follow, then finish, a read–D–formula–simplify solution.

  > 我能看懂并补全「读系数—算 D—代公式—化简」的解法。

  (8′)

- ○ I can solve one on my own, exact and to the nearest hundredth, with at most two hints.

  > 我能在最多两个提示下独立解一道，写出精确解和两位小数。

  (8′)

- ○ I can spot the five classic quadratic-formula mistakes.

  > 我能找出求根公式的五个经典错误。

  (8′)

- ○ I can do the Level 1–2 practice questions.

  > 我能做出 1–2 级练习题。

  (10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Work out $b^2 - 4ac$ when $a = 2$, $b = -3$ and $c = -5$.

> 当 $a = 2$、$b = -3$、$c = -5$ 时，求 $b^2 - 4ac$。

A. -31 B. 49 C. 31 D. -49

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

> 减去负数等于加上它：$5 - (-3) = 5 + 3 = 8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2 = 16$，但 $-4^2 = -16$（先算平方）。

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
   > $-7 - (-2)$：把「减负」改成「加」：$-7 + 2 = -5$。
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
   > $(-3)(-5) = 15$（同号），$(-3)(5) = -15$（异号）。
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.
   > 把 $x = -3$ 代入 x^2：先加括号，$(-3)^2 = 9$。

2. Simplify $\sqrt{28}$.

> 化简 $\sqrt{28}$。

A. $2\sqrt{7}$ B. $4\sqrt{7}$ C. $7\sqrt{2}$ D. 14

If you missed it · 如果做错了 — **Perfect squares and square roots**

> **完全平方数与平方根**

: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

> 记住 $1, 4, 9, \ldots, 225$ 这些平方数。$\sqrt{ab} = \sqrt{a}\sqrt{b}$，所以 $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$。但 $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$：$\sqrt{9+16} = 5$，不是 7。$\sqrt{x}$ 表示非负的那个根。

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
   > $\sqrt{72}$：最大的平方因数是 36 → $6\sqrt{2}$。
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.
   > $x^2 = 49$ 有两个解 $x = \pm 7$，但 $\sqrt{49}$ 只等于 7。

3. Simplify $\dfrac{6+4\sqrt{2}}{2}$.

> 化简 $\dfrac{6+4\sqrt{2}}{2}$。

A. $3+4\sqrt{2}$ B. $6+2\sqrt{2}$ C. $3+2\sqrt{2}$ D. $5\sqrt{2}$

If you missed it · 如果做错了 — **Fractions: add, subtract, multiply, divide**

> **分数的四则运算**

: Add or subtract only with a common denominator: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

> 加减必须先通分：$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$。乘法分子乘分子、分母乘分母。除以一个分数＝乘它的倒数（把第二个分数倒过来）。

1. $\frac{2}{3}-\frac{1}{4}$: common denominator 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$.
   > $\frac{2}{3}-\frac{1}{4}$：公分母 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$。
2. $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$.
   > $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$。
3. Never add denominators: $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$.
   > 分母不能相加：$\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$。

4. Write $2x^2=5x-1$ in the form $ax^2+bx+c=0$.

> 把 $2x^2=5x-1$ 写成 $ax^2+bx+c=0$ 的形式。

A. $2x^2+5x-1=0$ B. $2x^2-5x+1=0$ C. $2x^2-5x-1=0$ D. $-2x^2-5x+1=0$

If you missed it · 如果做错了 — **Rearranging a formula**

> **公式变形（求某个字母）**

: Treat the other letters as numbers and undo operations in reverse order. $A=P(1+r)$ for r: divide by P, then subtract 1: $r=\frac{A}{P}-1$.

> 把其他字母当成数字，按相反顺序撤销运算。由 $A=P(1+r)$ 求 r：先除以 P，再减 1：$r=\frac{A}{P}-1$。

1. $y=a(x-p)^2+q$ for a: subtract q, divide by $(x-p)^2$: $a=\frac{y-q}{(x-p)^2}$.
   > 由 $y=a(x-p)^2+q$ 求 a：减 q，再除以 $(x-p)^2$：$a=\frac{y-q}{(x-p)^2}$。
2. Substitute numbers first if the question gives them — fewer letters, fewer slips.
   > 题目给了数字就先代入——字母越少，越不容易错。

Answers · 答案: 1: B · 2: A · 3: C · 4: B

🔍 Common-errors sheet

常见错误单

1. Dividing only part of the top

> 分子只除了一部分

Solve $x^2 - 8x + 4 = 0$.

> 解 $x^2 - 8x + 4 = 0$。

1 $D = 64 - 16 = 48$

2 $x = \dfrac{8 \pm \sqrt{48}}{2} = \dfrac{8 \pm 4\sqrt{3}}{2}$

3 $x = 8 \pm 2\sqrt{3}$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Reading c before moving every term

> 还没移项就读 c

Solve $x^2 = 2x + 5$.

> 解 $x^2 = 2x + 5$。

1 $a = 1, b = 2, c = 5$

> $a = 1,\ b = 2,\ c = 5$

2 $D = 4 - 20 = -16$

3 No real solution.

> 无实数解。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Getting the sign of 4ac wrong when c is negative

> **c 为负时 4ac 的符号弄错**

Solve $x^2 + 2x - 6 = 0$.

> 解 $x^2 + 2x - 6 = 0$。

1 $D = 4 - 4(1)(6) = 4 - 24 = -20$

2 No real solution.

> 无实数解。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Starting with b instead of negative b

> **分子开头用了 b 而不是 –b**

Solve $x^2 - 5x + 3 = 0$.

> 解 $x^2 - 5x + 3 = 0$。

1 $D = 25 - 12 = 13$

2 $x = \dfrac{-5 \pm \sqrt{13}}{2}$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Dividing by a instead of 2a

> **分母用了 a 而不是 2a**

Solve $2x^2 - 6x + 1 = 0$.

> 解 $2x^2 - 6x + 1 = 0$。

1 $D = 36 - 8 = 28$

2 $x = \dfrac{6 \pm \sqrt{28}}{2}$

3 $x = 3 \pm \sqrt{7}$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 3 —** Only the radical part looked divisible, so only it was divided. The fraction bar covers the whole top.

   > 只觉得根式那项能约，就只除了它。分数线下面管的是整个分子。

   Divide both terms by 2: $x = 4 \pm 2\sqrt{3}$.

   > 两项都除以 2：$x = 4 \pm 2\sqrt{3}$。

2. **line · 第 1 —** The numbers were read off where they sit. The formula only works for $ax^2 + bx + c = 0$, so both terms must move left first, changing sign.

   > 直接照着位置读了数。公式只适用于 $ax^2 + bx + c = 0$，两项都要先移到左边并变号。

   $x^2 - 2x - 5 = 0$, $D = 4 + 20 = 24$, $x = \dfrac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}$.

   > $x^2 - 2x - 5 = 0$，$D = 4 + 20 = 24$，$x = \dfrac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}$。

3. **line · 第 1 —** The minus sign of c was left out. $-4(1)(-6)$ is $+24$, so D gets bigger, not smaller.

   > 漏了 c 的负号。$-4(1)(-6) = +24$，D 变大而不是变小。

   $D = 4 + 24 = 28$, $x = \dfrac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$.

   > $D = 4 + 24 = 28$，$x = \dfrac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$。

4. **line · 第 2 —** "$-b$" was read as "write a minus and then 5". Here $b = -5$, so $-b = +5$.

   > 把「$-b$」理解成「写个负号再写 5」。这里 $b = -5$，所以 $-b = +5$。

   $x = \dfrac{5 \pm \sqrt{13}}{2}$. Check: the answers add to $5 = -\frac{b}{a}$.

   > $x = \dfrac{5 \pm \sqrt{13}}{2}$。验算：两根之和为 $5 = -\frac{b}{a}$。

5. **line · 第 2 —** With $a = 2$ the bottom looked like "2" already. But the bottom is $2a = 2(2) = 4$.

   > $a = 2$，分母看起来已经是「2」了。可分母是 $2a = 2(2) = 4$。

   $x = \dfrac{6 \pm 2\sqrt{7}}{4} = \dfrac{3 \pm \sqrt{7}}{2}$.

   > $x = \dfrac{6 \pm 2\sqrt{7}}{4} = \dfrac{3 \pm \sqrt{7}}{2}$。