Solving by factoring & square roots

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Is one side already 0? Is there a ready-made square like $(x-3)^2$? Is there a common factor, such as an x in every term?
2. 🎯 **Asked** Every x that makes the equation true. A quadratic can have two, one or no real answers.
3. ✏️ **Picture it** The answers are where $y =$ left side meets $y =$ right side. With 0 on one side, they are the x-intercepts of the parabola.
4. 🧰 **Tool** Ready-made square: isolate it and take $\pm\sqrt{\ }$. Otherwise move everything to one side, factor, and set each factor to 0.
5. ✅ **Check** Put each answer back into the ORIGINAL equation. Two answers from $\pm$ should be equally far from the centre number.

⭐ Must know

- Zero product: get 0 on one side FIRST, then set each factor $= 0$.
- $(\ldots)^2 = k$: take $\pm\sqrt{k}$; $k < 0$ → no real solution.
- Never divide by x: factor it out, or the root $x = 0$ is lost.

$$AB = 0 \;\Rightarrow\; A = 0 \text{ or } B = 0$$

$$(x-p)^2 = k \;\Rightarrow\; x = p \pm \sqrt{k} \quad (k \geq 0)$$

🔍 Watch out for

1. Setting each factor equal to a number that is not zero
2. Dividing both sides by x
3. Taking only the positive square root
4. Undoing the number in front the wrong way
5. Taking the square root of a negative number

🌱 Small wins — tick as you go

- ○ I can factor $x^2 - x - 12$, solve $x^2 = 49$ with $\pm$, and simplify $\sqrt{18}$. (5′)
- ○ I can ask the five starting questions and choose between factoring and square roots. (5′)
- ○ I can follow, then finish, a make-it-zero, factor, split solution. (8′)
- ○ I can solve an equation like $x(x+5) = 24$ on my own with at most two hints. (7′)
- ○ I can spot the five classic mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Factor $x^2 - x - 12$.
A. $(x-4)(x+3)$ B. $(x+4)(x-3)$ C. $(x-6)(x+2)$ D. $(x-12)(x+1)$

If you missed it — **Factoring x^2+bx+c (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12$ → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x+2)(x+5)$.
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.

2. Solve $x^2 = 49$.
A. $x = 7$ B. $x = \pm 7$ C. $x = -7$ D. $x = 24.5$

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.

3. Simplify $\sqrt{18}$.
A. $3\sqrt{2}$ B. $9\sqrt{2}$ C. $2\sqrt{3}$ D. $6\sqrt{3}$

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.

4. Solve $3x + 4 = 0$.
A. $x = -\frac{4}{3}$ B. $x = \frac{4}{3}$ C. $x = -\frac{3}{4}$ D. $x = -12$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x-2) = x + 8$ → $3x - 6 = x + 8$ → $2x = 14$ → $x = 7$.
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30$ → $x = 6$.
3. Check by substituting: $3(7-2) = 15$ and $7 + 8 = 15$ ✓.

Answers: 1: A · 2: B · 3: A · 4: A

🔍 Common-errors sheet

1. Setting each factor equal to a number that is not zero

Solve $(x-1)(x+2)=10$.

1 $x-1=10$ or $x+2=10$

2 $x=11$ or $x=8$

✍ Which line goes wrong first, and why?

2. Dividing both sides by x

Solve $4x^2=28x$.

1 Divide both sides by $4x$: $x=7$

2 So the only solution is $x=7$.

✍ Which line goes wrong first, and why?

3. Taking only the positive square root

Solve $(x+3)^2=16$.

1 $x+3=4$

2 $x=1$

✍ Which line goes wrong first, and why?

4. Undoing the number in front the wrong way

Solve $2(x+1)^2 = 32$.

1 $(x+1)^2 = 30$

2 $x+1 = \pm\sqrt{30}$

3 $x = -1 \pm \sqrt{30}$

✍ Which line goes wrong first, and why?

5. Taking the square root of a negative number

Solve $(x-4)^2 + 9 = 0$.

1 $(x-4)^2 = -9$

2 $x-4 = \pm 3$

3 $x = 7$ or $x = 1$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1 —** The rule only works for 0: if a product is 0, one factor is 0. Two numbers can multiply to 10 in endless ways (2×5, 4×2.5, …). Expand and move 10: $x^2 + x - 12 = 0$, $(x+4)(x-3) = 0$, so $x = -4$ or $x = 3$.
2. **line 1 —** Dividing by x quietly assumes $x \neq 0$, and $x = 0$ is one of the answers: $4(0)^2 = 28(0)$. $4x^2 - 28x = 0$, $4x(x-7) = 0$, so $x = 0$ or $x = 7$.
3. **line 1 —** The $\sqrt{}$ key on a calculator gives only the positive root, but $(-4)^2$ is 16 too. $x + 3 = \pm 4$, so $x = 1$ or $x = -7$.
4. **line 1 —** The 2 MULTIPLIES the square, so it is undone by dividing by 2, not by subtracting 2. $(x+1)^2 = 16$, $x + 1 = \pm 4$, so $x = 3$ or $x = -5$.
5. **line 2 —** The minus sign was dropped to make the root work. No real number squared is negative. Stop at line 1: no real solution. (Check: $(7-4)^2 + 9 = 18 \neq 0$.)