# Solving by factoring & square roots

## 因式分解法与直接开平方法

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

### 🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Is one side already 0? Is there a ready-made square like $(x-3)^2$? Is there a common factor, such as an $x$ in every term?
   > 有没有一边已经是 0？有没有现成的平方，比如 $(x-3)^2$？每一项有没有公因式，比如都带 $x$？
2. 🎯 **Asked · 求什么** Every $x$ that makes the equation true. A quadratic can have two, one or no real answers.
   > 让方程成立的所有 $x$。二次方程可能有两个、一个或没有实数解。
3. ✏️ **Picture it · 画出来** The answers are where $y =$ left side meets $y =$ right side. With 0 on one side, they are the x-intercepts of the parabola.
   > 答案就是 $y =$左边与 $y =$右边的交点。一边是 0 时，答案就是抛物线的 x 截距。
4. 🧰 **Tool · 用什么工具** Ready-made square: isolate it and take $\pm\sqrt{\ }$. Otherwise move everything to one side, factor, and set each factor to 0.
   > 有现成的平方：先把它单独留下，再开方取 ±。否则把所有项移到一边，分解，再令每个因式等于 0。
5. ✅ **Check · 检验** Put each answer back into the ORIGINAL equation. Two answers from ± should be equally far from the centre number.
   > 把每个答案代回「原」方程。± 得到的两个答案，到中间那个数的距离应该相等。

### ⭐ Must know

必须记住

- Zero product: get $0$ on one side FIRST, then set each factor $= 0$.
  > 零积法则：先让一边等于 $0$，再令每个因式 $= 0$。
- $(\ldots)^2 = k$: take $\pm\sqrt{k}$; $k < 0$ → no real solution.
  > $(\ldots)^2 = k$：开方取 $\pm\sqrt{k}$；$k < 0$ → 无实数解。
- Never divide by $x$: factor it out, or the root $x = 0$ is lost.
  > 不要两边除以 $x$：把 $x$ 提出来，否则丢掉根 $x = 0$。

$$AB = 0 \;\Rightarrow\; A = 0 \text{ or } B = 0$$

$$(x-p)^2 = k \;\Rightarrow\; x = p \pm \sqrt{k} \quad (k \geq 0)$$

### 🔍 Watch out for

当心

1. Setting each factor equal to a number that is not zero
   > 让因式等于一个不是零的数
2. Dividing both sides by x
   > 两边同除以 x
3. Taking only the positive square root
   > 开平方只取了正根
4. Undoing the number in front the wrong way
   > 前面的系数还原错了
5. Taking the square root of a negative number
   > 对负数开平方

## 🌱 Small wins — tick as you go

一小步一小步——做完打勾

- ○ I can factor $x^2 - x - 12$, solve $x^2 = 49$ with $\pm$, and simplify $\sqrt{18}$.
  > 我会分解 $x^2 - x - 12$，会用 $\pm$ 解 $x^2 = 49$，也会化简 $\sqrt{18}$。

  (5′)
- ○ I can ask the five starting questions and choose between factoring and square roots.
  > 我能问出五个入手问题，并在分解法和开平方法之间做选择。

  (5′)
- ○ I can follow, then finish, a make-it-zero, factor, split solution.
  > 我能看懂并补全「化成 0—分解—拆开」的解法。

  (8′)
- ○ I can solve an equation like $x(x + 5) = 24$ on my own with at most two hints.
  > 我能在最多两个提示下独立解 $x(x + 5) = 24$ 这类方程。

  (7′)
- ○ I can spot the five classic mistakes in someone else's work.
  > 我能在别人的解答里找出五个经典错误。

  (8′)
- ○ I can do the Level 1–2 practice questions.
  > 我能做出 1–2 级练习题。

  (10′)

# Prerequisite repair sheet

## 先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

**1.** Factor $x^2 - x - 12$.

> 分解 $x^2 - x - 12$。

A. $(x-4)(x+3)$ B. $(x+4)(x-3)$ C. $(x-6)(x+2)$ D. $(x-12)(x+1)$

If you missed it · 如果做错了 — **Factoring $x^2 + bx + c$ (product and sum)**

> **分解 $x^2 + bx + c$（积与和）**

: Take out a common factor first. Then find two numbers that MULTIPLY to $c$ and ADD to $b$: $x^2 - x - 12$ → $-4$ and $3$ → $(x-4)(x+3)$. Check by expanding.

> 先提公因式。再找两个数，相乘等于 $c$、相加等于 $b$：$x^2 - x - 12$ → $-4$ 和 $3$ → $(x-4)(x+3)$。展开检验。

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x+2)(x+5)$.
   > $x^2 + 7x + 10$：$2 \times 5 = 10$，$2 + 5 = 7$ → $(x+2)(x+5)$。
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of $b$.
   > 若 $c < 0$，两个数异号；若 $c > 0$，两个数与 $b$ 同号。

**2.** Solve $x^2 = 49$.

> 解 $x^2 = 49$。

A. $x = 7$ B. $x = \pm 7$ C. $x = -7$ D. $x = 24.5$

If you missed it · 如果做错了 — **Perfect squares and square roots**

> **完全平方数与平方根**

: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

> 记住 $1, 4, 9, \ldots, 225$ 这些平方数。$\sqrt{ab} = \sqrt{a}\sqrt{b}$，所以 $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$。但 $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$：$\sqrt{9+16} = 5$，不是 7。$\sqrt{x}$ 表示非负的那个根。

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
   > $\sqrt{72}$：最大的平方因数是 36 → $6\sqrt{2}$。
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.
   > $x^2 = 49$ 有两个解 $x = \pm 7$，但 $\sqrt{49}$ 只等于 7。

**3.** Simplify $\sqrt{18}$.

> 化简 $\sqrt{18}$。

A. $3\sqrt{2}$ B. $9\sqrt{2}$ C. $2\sqrt{3}$ D. $6\sqrt{3}$

If you missed it · 如果做错了 — **Perfect squares and square roots**

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**4.** Solve $3x + 4 = 0$.

> 解 $3x + 4 = 0$。

A. $x = -\frac{4}{3}$ B. $x = \frac{4}{3}$ C. $x = -\frac{3}{4}$ D. $x = -12$

If you missed it · 如果做错了 — **Solving a linear equation**

> **解一元一次方程**

: Do the same thing to both sides. Clear brackets, collect $x$ on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

> 两边做同样的事。先去括号，把含 $x$ 的项移到一边、数字移到另一边，最后除以系数。有分数时先把每一项都乘以公分母。

1. $3(x - 2) = x + 8$ → $3x - 6 = x + 8$ → $2x = 14$ → $x = 7$.
   > $3(x - 2) = x + 8$ → $3x - 6 = x + 8$ → $2x = 14$ → $x = 7$。
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30$ → $x = 6$.
   > $\frac{x}{2} + \frac{x}{3} = 5$：每项乘 6 → $3x + 2x = 30$ → $x = 6$。
3. Check by substituting: $3(7 - 2) = 15$ and $7 + 8 = 15$ ✓.
   > 代回检验：$3(7 - 2) = 15$，$7 + 8 = 15$ ✓。

Answers · 答案: 1: A · 2: B · 3: A · 4: A

# 🔍 Common-errors sheet

## 常见错误单

### 1. Setting each factor equal to a number that is not zero

> **让因式等于一个不是零的数**

Solve $(x-1)(x+2)=10$.

> 解 $(x-1)(x+2)=10$。

**1** $x-1=10$ or $x+2=10$

> $x-1=10$ 或 $x+2=10$

**2** $x=11$ or $x=8$

> $x=11$ 或 $x=8$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

### 2. Dividing both sides by x

> **两边同除以 x**

Solve $4x^2=28x$.

> 解 $4x^2=28x$。

**1** Divide both sides by $4x$: $x=7$

> 两边同除以 $4x$：$x=7$

**2** So the only solution is $x=7$.

> 所以唯一的解是 $x=7$。

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

### 3. Taking only the positive square root

> **开平方只取了正根**

Solve $(x+3)^2=16$.

> 解 $(x+3)^2=16$。

**1** $x+3=4$

**2** $x=1$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

### 4. Undoing the number in front the wrong way

> **前面的系数还原错了**

Solve $2(x+1)^2 = 32$.

> 解 $2(x+1)^2 = 32$。

**1** $(x+1)^2 = 30$

**2** $x + 1 = \pm\sqrt{30}$

**3** $x = -1 \pm \sqrt{30}$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

### 5. Taking the square root of a negative number

> **对负数开平方**

Solve $(x-4)^2 + 9 = 0$.

> 解 $(x-4)^2 + 9 = 0$。

**1** $(x-4)^2 = -9$

**2** $x - 4 = \pm 3$

**3** $x = 7$ or $x = 1$

> $x = 7$ 或 $x = 1$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

**Key (fold this part back) · 答案（可以折起来）**

1. **line · 第 1 —** The rule only works for 0: if a product is 0, one factor is 0. Two numbers can multiply to 10 in endless ways ($2 \times 5$, $4 \times 2.5$, …).
   > 这条规则只对 0 成立：乘积为 0，才有一个因式是 0。两个数相乘等于 10 的方式有无数种（$2 \times 5$、$4 \times 2.5$……）。

   Expand and move 10: $x^2 + x - 12 = 0$, $(x + 4)(x - 3) = 0$, so $x = -4$ or $x = 3$.
   > 展开并移项：$x^2 + x - 12 = 0$，$(x + 4)(x - 3) = 0$，得 $x = -4$ 或 $x = 3$。
2. **line · 第 1 —** Dividing by $x$ quietly assumes $x \neq 0$, and $x = 0$ is one of the answers: $4(0)^2 = 28(0)$.
   > 除以 $x$ 时默认了 $x \neq 0$，可 $x = 0$ 正是一个解：$4(0)^2 = 28(0)$。

   $4x^2 - 28x = 0$, $4x(x - 7) = 0$, so $x = 0$ or $x = 7$.
   > $4x^2 - 28x = 0$，$4x(x - 7) = 0$，得 $x = 0$ 或 $x = 7$。
3. **line · 第 1 —** The $\sqrt{}$ key on a calculator gives only the positive root, but $(-4)^2$ is 16 too.
   > 计算器的 $\sqrt{}$ 键只给正根，可 $(-4)^2$ 也等于 16。

   $x + 3 = \pm 4$, so $x = 1$ or $x = -7$.
   > $x + 3 = \pm 4$，得 $x = 1$ 或 $x = -7$。
4. **line · 第 1 —** The 2 MULTIPLIES the square, so it is undone by dividing by 2, not by subtracting 2.
   > 2 是「乘」在平方上的，要用除以 2 来还原，不是减 2。

   $(x + 1)^2 = 16$, $x + 1 = \pm 4$, so $x = 3$ or $x = -5$.
   > $(x + 1)^2 = 16$，$x + 1 = \pm 4$，得 $x = 3$ 或 $x = -5$。
5. **line · 第 2 —** The minus sign was dropped to make the root work. No real number squared is negative.
   > 为了能开方，把负号丢掉了。没有哪个实数的平方是负数。

   Stop at line 1: no real solution. (Check: $(7 - 4)^2 + 9 = 18 \neq 0$.)
   > 第 1 行就停：无实数解。（检验：$(7 - 4)^2 + 9 = 18 \neq 0$。）