The discriminant

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Put the equation in the form $ax^2 + bx + c = 0$ (everything on one side), then write a, b, c with their signs. If there is a k, note which letter it lives in.
2. 🎯 **Asked** "How many roots" or "which k"? Equal roots → $D = 0$; two different → $D > 0$; none → $D < 0$; "real roots" (at least one) → $D \geq 0$.
3. ✏️ **Picture it** The parabola and the x-axis: it crosses twice ($D > 0$), just touches at the vertex ($D = 0$), or misses ($D < 0$).
4. 🧰 **Tool** $D = b^2 - 4ac$, with brackets around a negative b. With k, write D in terms of k, then solve the equation or inequality.
5. ✅ **Check** Put your k back: $D = 0$ should give a perfect square. For an inequality, test one k on each side. If k multiplies x^2, make sure $k \neq 0$.

⭐ Must know

- $D = b^2 - 4ac$: $D > 0$ two roots, $D = 0$ one (double) root, $D < 0$ no real roots.
- D a perfect square → rational roots → it factors.
- Unknown k: write D in k, set $= 0$, > 0 or < 0, solve.

$$D = b^2 - 4ac$$

🔍 Watch out for

1. Calling a zero discriminant no solution
2. Losing the sign when squaring a negative b
3. Using only part of the bracket as b
4. Forgetting that a cannot be zero
5. Not flipping the inequality for a negative

🌱 Small wins — tick as you go

- ○ I can square a negative number, solve $k^2 = 81$, and flip an inequality when I divide by a negative. (5′)
- ○ I can ask the five starting questions for a discriminant problem. (5′)
- ○ I can follow, then finish, a "write D in k, set $D = 0$, check" solution. (8′)
- ○ I can find k on my own when k sits in front of x^2, with at most two hints. (7′)
- ○ I can spot the five classic discriminant mistakes. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Evaluate $b^2 - 4ac$ when $a = 1$, $b = -4$, $c = -5$.
A. -4 B. 4 C. 36 D. -36

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

2. What is $(-7)^2$?
A. -49 B. 49 C. 14 D. -14

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

3. Solve $k^2 = 81$.
A. $k = \pm 9$ B. $k = 9$ C. $k = \pm 3$ D. $k = 40.5$

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.

4. Solve $-3k > 12$.
A. $k > -4$ B. $k > 4$ C. $k < 4$ D. $k < -4$

If you missed it — **Inequalities: when the sign flips**: Solve like an equation, EXCEPT: multiplying or dividing both sides by a negative number flips the sign. $-2x > 6$ → $x < -3$. Test a number to be sure: $x = -4$ gives $8 > 6$ ✓.

1. $5 - 3x \leq 11$ → $-3x \leq 6$ → $x \geq -2$ (divided by -3: flip).
2. Number line: closed dot for $\leq, \geq$; open dot for $<, >$.

Answers: 1: C · 2: B · 3: A · 4: D

🔍 Common-errors sheet

1. Calling a zero discriminant no solution

State the nature of the roots of $x^2 - 10x + 25 = 0$.

1 $D = (-10)^2 - 4(1)(25) = 100 - 100 = 0$

2 $D = 0$, so there are no real roots.

✍ Which line goes wrong first, and why?

2. Losing the sign when squaring a negative b

State the nature of the roots of $2x^2 - 6x + 3 = 0$.

1 $D = -6^2 - 4(2)(3) = -36 - 24 = -60$

2 $D < 0$, so no real roots.

✍ Which line goes wrong first, and why?

3. Using only part of the bracket as b

Find k so that $x^2 + (k + 1)x + 9 = 0$ has two equal real roots.

1 $b = k$, so $D = k^2 - 36$

2 $k^2 - 36 = 0$, so $k = \pm 6$.

✍ Which line goes wrong first, and why?

4. Forgetting that a cannot be zero

For which k does $kx^2 + 4x + 1 = 0$ have two different real roots?

1 $D = 16 - 4k$

2 $16 - 4k > 0$, so $k < 4$

3 Answer: $k < 4$.

✍ Which line goes wrong first, and why?

5. Not flipping the inequality for a negative

For which k does $x^2 + 6x + k = 0$ have no real roots?

1 $D = 36 - 4k < 0$

2 $-4k < -36$

3 $k < 9$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** "Zero" sounds like "nothing". But $\sqrt{0} = 0$, so $\pm\sqrt{D}$ gives the same number twice: one repeated root. $D = 0$: two equal real roots. In fact $(x - 5)^2 = 0$, so $x = 5$.
2. **line 1 —** Without brackets, -6^2 means $-(6^2) = -36$. The whole $b = -6$ must be squared: $(-6)^2 = 36$. A square is never negative. $D = 36 - 24 = 12 > 0$: two different real roots (irrational, since 12 is not a perfect square).
3. **line 1 —** b is everything in front of x, so $b = k + 1$. Squaring only k gives a different equation. $D = (k + 1)^2 - 36 = 0 \Rightarrow k + 1 = \pm 6 \Rightarrow k = 5$ or $k = -7$.
4. **line 3 —** $k = 0$ is less than 4, but then the equation is $4x + 1 = 0$, which is linear and has only one root. Keep $D > 0$ and add $k \neq 0$: $k < 4$ and $k \neq 0$.
5. **line 3 —** Dividing both sides by -4 reverses the inequality. A quick test shows it: $k = 0$ gives $D = 36 > 0$, two roots, so $k < 9$ cannot be right. $-4k < -36 \Rightarrow k > 9$. Test $k = 10$: $D = 36 - 40 = -4 < 0$ ✓.