Choosing a method & word problems

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Read it twice and underline every number with what it describes. Let the variable be the thing asked for, then write every other quantity using it (width w, length $w + 5$).
2. 🎯 **Asked** Which quantity, in which unit? One value, a pair of numbers, or every time something happens (a ball can pass a height twice)?
3. ✏️ **Picture it** Sketch it: a rectangle with the border labelled x on BOTH sides, a right triangle with its three sides labelled, or a height-vs-time arc with a line at the target height.
4. 🧰 **Tool** Write ONE equation from ONE fact: area = length × width, $a^2 + b^2 = c^2$, or $h(t) =$ the height. Move everything to one side; if D is a perfect square, factor, otherwise use the formula.
5. ✅ **Check** Test each root against the WORDS: negative lengths and times go, with a reason. Put the kept answer back into the story, then finish with a sentence and units.

⭐ Must know

- Variable = what is asked; equation from area, Pythagoras or "height = value".
- A frame or path of width x adds $2x$ to each dimension.
- Solve fully, then reject roots that make no sense (say why); keep both if both fit.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a^2 + b^2 = c^2$$

🔍 Watch out for

1. Adding the border width only once
2. Using the zero rule when the right side is not zero
3. Throwing away a valid second time
4. Solving for the ground instead of the height asked
5. Keeping a negative length

🌱 Small wins — tick as you go

- ○ I can expand two binomials, factor a trinomial and use Pythagoras without notes. (5′)
- ○ I can name the variable and write every other quantity in terms of it before touching any algebra. (7′)
- ○ I can follow, then finish, a border problem: sketch, equation, solve, reject. (10′)
- ○ I can turn a new word problem into one equation with at most two hints. (8′)
- ○ I can spot the five classic word-problem mistakes in someone else's work. (10′)
- ○ I can do the Level 1–2 practice questions and finish each with a sentence and units. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Expand $(6+2x)(4+2x)$.
A. $4x^2+24$ B. $4x^2+20x+24$ C. $2x^2+20x+24$ D. $4x^2+10x+24$

If you missed it — **Multiplying binomials,** $(a+b)^2$: Every term times every term (FOIL): $(x+3)(x-5)=x^2-5x+3x-15=x^2-2x-15$. A square has a middle term: $(x+3)^2=x^2+6x+9$, never x^2+9.

1. $(x-4)^2=(x-4)(x-4)=x^2-4x-4x+16=x^2-8x+16$.
2. Shortcut: $(a\pm b)^2=a^2\pm 2ab+b^2$ — middle term $=2\times$ first $\times$ last.

2. Solve $x^2+5x-24=0$.
A. $x=3$ or $x=-8$ B. $x=-3$ or $x=8$ C. $x=4$ or $x=-6$ D. $x=2$ or $x=-12$

If you missed it — **Factoring x^2+bx+c (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: x^2-x-12 → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2+7x+10$: $2\times 5=10$, $2+5=7$ → $(x+2)(x+5)$.
2. If $c<0$ the two numbers have different signs; if $c>0$ they share the sign of b.

3. A right triangle has legs 9 cm and 12 cm. How long is the hypotenuse?
A. 21 cm B. $\sqrt{63}$ cm C. 15 cm D. 225 cm

If you missed it — **SOH CAH TOA and Pythagoras**: In a RIGHT triangle, from angle θ: $\sin\theta=\frac{\text{opp}}{\text{hyp}}$, $\cos\theta=\frac{\text{adj}}{\text{hyp}}$, $\tan\theta=\frac{\text{opp}}{\text{adj}}$. The hypotenuse is opposite the right angle; $a^2+b^2=c^2$. Calculator in DEGREE mode.

1. Legs 3 and 4: hypotenuse $\sqrt{9+16}=5$; $\sin\theta=\frac{3}{5}$ for the angle opposite 3.
2. Find an angle: $\theta=\sin^{-1}(0.6)\approx 36.9^\circ$.

4. A rectangle is w cm wide and 4 cm longer than it is wide. Its perimeter is 36 cm. Find w.
A. $w=8$ B. $w=9$ C. $w=16$ D. $w=7$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x-2)=x+8$ → $3x-6=x+8$ → $2x=14$ → $x=7$.
2. $\frac{x}{2}+\frac{x}{3}=5$: multiply every term by 6 → $3x+2x=30$ → $x=6$.
3. Check by substituting: $3(7-2)=15$ and $7+8=15$ ✓.

Answers: 1: B · 2: A · 3: C · 4: D

🔍 Common-errors sheet

1. Adding the border width only once

A 15 cm by 9 cm photo has a frame of uniform width; together they cover 247 cm². Find the frame width.

1 $(15 + x)(9 + x) = 247$

2 $x^2 + 24x - 112 = 0$

3 $(x + 28)(x - 4) = 0$, so the frame is 4 cm.

✍ Which line goes wrong first, and why?

2. Using the zero rule when the right side is not zero

Two consecutive odd integers have a product of 143. Find them.

1 Let them be n and $n + 2$: $n(n + 2) = 143$

2 So $n = 143$ or $n + 2 = 143$

3 The numbers are 141 and 143.

✍ Which line goes wrong first, and why?

3. Throwing away a valid second time

A ball's height is $h(t) = -5t^2 + 30t + 1$ (m, s). When is it 26 m high?

1 $-5t^2 + 30t - 25 = 0 \Rightarrow t^2 - 6t + 5 = 0$

2 $(t - 1)(t - 5) = 0$, $t = 1$ or $t = 5$

3 It is 26 m high at $t = 1$ s.

✍ Which line goes wrong first, and why?

4. Solving for the ground instead of the height asked

A ball's height is $h(t) = -5t^2 + 35t + 1$ (m, s). When is it 51 m high?

1 Set $-5t^2 + 35t + 1 = 0$

2 Formula: $t \approx 7.03$ or $t \approx -0.03$

3 It is 51 m high at about 7.03 s.

✍ Which line goes wrong first, and why?

5. Keeping a negative length

One leg of a right triangle is 2 cm longer than the other, and the hypotenuse is 10 cm. Find the legs.

1 $x^2 + (x+2)^2 = 100 \Rightarrow x^2 + 2x - 48 = 0$

2 $(x+8)(x-6) = 0$, $x = 6$ or $x = -8$

3 Legs 6 and 8, or -8 and -6.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1 —** The picture in your head had the frame on one side. It sits on the left AND right, top AND bottom, so each dimension grows by $2x$. $(15+2x)(9+2x) = 247 \Rightarrow 4x^2 + 48x - 112 = 0 \Rightarrow x^2 + 12x - 28 = 0 \Rightarrow (x+14)(x-2) = 0$; $x = 2$. Check $19 \times 13 = 247$ ✓.
2. **line 2 —** "One factor must be 0" only works when the product IS 0. A product of 143 tells you nothing about each factor on its own. Expand and move: $n^2 + 2n - 143 = 0$, $(n+13)(n-11) = 0$, $n = 11$ or $n = -13$. Check $11 \times 13 = 143$ ✓.
3. **line 3 —** The habit "a word problem has one answer" kicks in. But both times are positive: the ball passes 26 m going up and again coming down. Keep every root that fits the story: $t = 1$ s and $t = 5$ s (the axis $t = 3$ sits halfway between them ✓).
4. **line 1 —** "Set it equal to 0" became automatic. Zero means the ground; this question asks for a height of 51, so $h(t)$ must equal 51. $-5t^2 + 35t + 1 = 51 \Rightarrow -5t^2 + 35t - 50 = 0 \Rightarrow t^2 - 7t + 10 = 0 \Rightarrow (t-2)(t-5) = 0$: $t = 2$ s and $t = 5$ s.
5. **line 3 —** The equation does not know x is a length, so it hands back both roots. Your job is to put the story back in. A length cannot be -8: reject it. Legs 6 cm and 8 cm; check $36 + 64 = 100$ ✓.