Choosing a method & word problems

选方法与应用题

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Read it twice and underline every number with what it describes. Let the variable be the thing asked for, then write every other quantity using it (width w, length $w + 5$).

   > 读两遍，把每个数字和它描述的东西划出来。设所求的量为未知数，再用它表示其他量（宽 w，长 $w + 5$）。

2. 🎯 **Asked · 求什么** Which quantity, in which unit? One value, a pair of numbers, or every time something happens (a ball can pass a height twice)?

   > 求哪个量、什么单位？要一个值、一对数，还是「所有时刻」（球可能两次经过同一高度）？

3. ✏️ **Picture it · 画出来** Sketch it: a rectangle with the border labelled x on BOTH sides, a right triangle with its three sides labelled, or a height-vs-time arc with a line at the target height.

   > 画个图：矩形四周的边框「两侧」都标上 x；直角三角形三边都标好；或者画高度—时间的弧线，再画一条目标高度的水平线。

4. 🧰 **Tool · 用什么工具** Write ONE equation from ONE fact: area = length × width, $a^2 + b^2 = c^2$, or $h(t) =$ the height. Move everything to one side; if D is a perfect square, factor, otherwise use the formula.

   > 用「一个事实」列「一个方程」：面积 = 长 × 宽、$a^2 + b^2 = c^2$，或 $h(t) =$ 某高度。全部移到一边；D 是完全平方数就分解，否则用公式。

5. ✅ **Check · 检验** Test each root against the WORDS: negative lengths and times go, with a reason. Put the kept answer back into the story, then finish with a sentence and units.

   > 每个根都拿回「题目原文」检验：负的长度、时间要写理由舍去。把留下的答案代回情境验算，最后写一句带单位的话。

⭐ Must know

必须记住

- Variable = what is asked; equation from area, Pythagoras or "height = value".

  > 设所求为未知数；由面积、勾股定理或「高度 = 某值」列方程。

- A frame or path of width x adds $2x$ to each dimension.

  > 宽为 x 的边框或小路，使每个方向的长度都增加 $2x$。

- Solve fully, then reject roots that make no sense (say why); keep both if both fit.

  > 先完整解出，再舍去不合理的根（写出理由）；两个都合理就都保留。

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a^2 + b^2 = c^2$$

🔍 Watch out for

当心

1. Adding the border width only once

   > 边框宽只加了一次

2. Using the zero rule when the right side is not zero

   > 右边不是 0 就用零积法则

3. Throwing away a valid second time

   > 扔掉了有效的第二个时刻

4. Solving for the ground instead of the height asked

   > 求成了落地时间而不是题目问的高度

5. Keeping a negative length

   > 留下了负的长度

🌱 Small wins — tick as you go

一小步一小步——做完打勾

○ I can expand two binomials, factor a trinomial and use Pythagoras without notes.

> 我能不看笔记展开两个二项式、分解三项式、用勾股定理。

(5′)

○ I can name the variable and write every other quantity in terms of it before touching any algebra.

> 我能在动笔算之前先设好未知数，并用它表示出其他所有量。

(7′)

○ I can follow, then finish, a border problem: sketch, equation, solve, reject.

> 我能看懂并补全一道边框题：画图、列方程、求解、舍根。

(10′)

○ I can turn a new word problem into one equation with at most two hints.

> 我能在最多两个提示下，把一道新应用题变成一个方程。

(8′)

○ I can spot the five classic word-problem mistakes in someone else's work.

> 我能在别人的解答里找出应用题的五个经典错误。

(10′)

○ I can do the Level 1–2 practice questions and finish each with a sentence and units.

> 我能做出 1–2 级练习题，并且每题最后写一句带单位的结论。

(10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Expand $(6+2x)(4+2x)$.

展开 $(6+2x)(4+2x)$。

A. $4x^2+24$ B. $4x^2+20x+24$ C. $2x^2+20x+24$ D. $4x^2+10x+24$

If you missed it · 如果做错了 — **Multiplying binomials, $(a+b)^2$**

二项式相乘与 $(a+b)^2$

: Every term times every term (FOIL): $(x+3)(x-5)=x^2-5x+3x-15=x^2-2x-15$. A square has a middle term: $(x+3)^2=x^2+6x+9$, never x^2+9.

每一项乘每一项：$(x+3)(x-5)=x^2-5x+3x-15=x^2-2x-15$。平方会有中间项：$(x+3)^2=x^2+6x+9$，绝不是 x^2+9。

1. $(x-4)^2=(x-4)(x-4)=x^2-4x-4x+16=x^2-8x+16$.
   $(x-4)^2=(x-4)(x-4)=x^2-4x-4x+16=x^2-8x+16$。
2. Shortcut: $(a\pm b)^2=a^2\pm 2ab+b^2$ — middle term $=2\times$ first $\times$ last.
   捷径：$(a\pm b)^2=a^2\pm 2ab+b^2$——中间项=2×首项×末项。

2. Solve $x^2+5x-24=0$.

解 $x^2+5x-24=0$。

A. $x=3$ or $x=-8$ B. $x=-3$ or $x=8$ C. $x=4$ or $x=-6$ D. $x=2$ or $x=-12$

If you missed it · 如果做错了 — **Factoring x^2+bx+c (product and sum)**

分解 x^2+bx+c（积与和）

: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: x^2-x-12 → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

先提公因式。再找两个数，相乘等于 c、相加等于 b：x^2-x-12 → -4 和 3 → $(x-4)(x+3)$。展开检验。

1. $x^2+7x+10$: $2\times 5=10$, $2+5=7$ → $(x+2)(x+5)$.
   $x^2+7x+10$：$2\times 5=10$，$2+5=7$ → $(x+2)(x+5)$。
2. If $c<0$ the two numbers have different signs; if $c>0$ they share the sign of b.
   若 $c<0$，两个数异号；若 $c>0$，两个数与 b 同号。

3. A right triangle has legs 9 cm and 12 cm. How long is the hypotenuse?

直角三角形两条直角边是 9 cm 和 12 cm，斜边多长?

A. 21 cm B. $\sqrt{63}$ cm C. 15 cm D. 225 cm

If you missed it · 如果做错了 — **SOH CAH TOA and Pythagoras**

直角三角形：SOH CAH TOA 与勾股定理

: In a RIGHT triangle, from angle θ: $\sin\theta=\frac{\text{opp}}{\text{hyp}}$, $\cos\theta=\frac{\text{adj}}{\text{hyp}}$, $\tan\theta=\frac{\text{opp}}{\text{adj}}$. The hypotenuse is opposite the right angle; $a^2+b^2=c^2$. Calculator in DEGREE mode.

在直角三角形中，从角 θ 看：$\sin\theta=\frac{\text{对边}}{\text{斜边}}$，$\cos\theta=\frac{\text{邻边}}{\text{斜边}}$，$\tan\theta=\frac{\text{对边}}{\text{邻边}}$。斜边对着直角；$a^2+b^2=c^2$。计算器设为角度（DEG）模式。

1. Legs 3 and 4: hypotenuse $\sqrt{9+16}=5$; $\sin\theta=\frac{3}{5}$ for the angle opposite 3.
   两直角边 3 和 4：斜边 $\sqrt{9+16}=5$；对着 3 的角 $\sin\theta=\frac{3}{5}$。
2. Find an angle: $\theta=\sin^{-1}(0.6)\approx 36.9^\circ$.
   求角：$\theta=\sin^{-1}(0.6)\approx 36.9^\circ$。

4. A rectangle is w cm wide and 4 cm longer than it is wide. Its perimeter is 36 cm. Find w.

> 一个长方形宽 w cm，长比宽多 4 cm，周长 36 cm。求 w。

A. $w = 8$ B. $w = 9$ C. $w = 16$ D. $w = 7$

If you missed it · 如果做错了 — **Solving a linear equation**

> **解一元一次方程**

: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

> 两边做同样的事。先去括号，把含 x 的项移到一边、数字移到另一边，最后除以系数。有分数时先把每一项都乘以公分母。

1. $3(x-2) = x+8$ → $3x-6 = x+8$ → $2x = 14$ → $x = 7$.
   > $3(x-2) = x+8$ → $3x-6 = x+8$ → $2x = 14$ → $x = 7$。
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30$ → $x = 6$.
   > $\frac{x}{2} + \frac{x}{3} = 5$：每项乘 6 → $3x + 2x = 30$ → $x = 6$。
3. Check by substituting: $3(7-2) = 15$ and $7+8 = 15$ ✓.
   > 代回检验：$3(7-2) = 15$，$7+8 = 15$ ✓。

Answers · 答案: 1: B · 2: A · 3: C · 4: D

🔍 Common-errors sheet

常见错误单

1. Adding the border width only once

> **边框宽只加了一次**

A 15 cm by 9 cm photo has a frame of uniform width; together they cover 247 cm². Find the frame width.

> 一张 15 cm × 9 cm 的照片配等宽相框，合计 247 cm²。求相框宽。

1 $(15 + x)(9 + x) = 247$

2 $x^2 + 24x - 112 = 0$

3 $(x + 28)(x - 4) = 0$, so the frame is 4 cm.

> $(x + 28)(x - 4) = 0$，所以相框宽 4 cm。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Using the zero rule when the right side is not zero

> **右边不是 0 就用零积法则**

Two consecutive odd integers have a product of 143. Find them.

> 两个连续奇数的积是 143，求这两个数。

1 Let them be n and $n + 2$: $n(n + 2) = 143$

> 设两数为 n 和 $n + 2$：$n(n + 2) = 143$

2 So $n = 143$ or $n + 2 = 143$

> 所以 $n = 143$ 或 $n + 2 = 143$

3 The numbers are 141 and 143.

> 两数是 141 和 143。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Throwing away a valid second time

> **扔掉了有效的第二个时刻**

A ball's height is $h(t) = -5t^2 + 30t + 1$ (m, s). When is it 26 m high?

> 球的高度为 $h(t) = -5t^2 + 30t + 1$（米，秒）。什么时候高 26 m?

1 $-5t^2 + 30t - 25 = 0 \Rightarrow t^2 - 6t + 5 = 0$

2 $(t-1)(t-5) = 0$, $t = 1$ or $t = 5$

> $(t-1)(t-5) = 0$，$t = 1$ 或 $t = 5$

3 It is 26 m high at $t = 1$ s.

> 球在 $t = 1$ 秒时高 26 m。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Solving for the ground instead of the height asked

> **求成了落地时间而不是题目问的高度**

A ball's height is $h(t) = -5t^2 + 35t + 1$ (m, s). When is it 51 m high?

> 球的高度为 $h(t) = -5t^2 + 35t + 1$（米，秒）。什么时候高 51 m?

1 Set $-5t^2 + 35t + 1 = 0$

> 令 $-5t^2 + 35t + 1 = 0$

2 Formula: $t \approx 7.03$ or $t \approx -0.03$

> 公式：$t \approx 7.03$ 或 $t \approx -0.03$

3 It is 51 m high at about 7.03 s.

> 约 7.03 秒时高 51 m。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Keeping a negative length

> **留下了负的长度**

One leg of a right triangle is 2 cm longer than the other, and the hypotenuse is 10 cm. Find the legs.

> 直角三角形一条直角边比另一条长 2 cm，斜边 10 cm。求两条直角边。

1 $x^2 + (x+2)^2 = 100 \Rightarrow x^2 + 2x - 48 = 0$

2 $(x+8)(x-6) = 0$, $x = 6$ or $x = -8$

> $(x+8)(x-6) = 0$，$x = 6$ 或 $x = -8$

3 Legs 6 and 8, or -8 and -6.

> 直角边 6 和 8，或 -8 和 -6。

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 1 —** The picture in your head had the frame on one side. It sits on the left AND right, top AND bottom, so each dimension grows by $2x$.

   > 脑子里的图只有一侧有框。其实左右、上下都有，所以每个方向都增加 $2x$。

   $(15+2x)(9+2x) = 247 \Rightarrow 4x^2 + 48x - 112 = 0 \Rightarrow x^2 + 12x - 28 = 0 \Rightarrow (x+14)(x-2) = 0$; $x = 2$. Check $19 \times 13 = 247$ ✓.

   > $(15+2x)(9+2x) = 247 \Rightarrow 4x^2 + 48x - 112 = 0 \Rightarrow x^2 + 12x - 28 = 0 \Rightarrow (x+14)(x-2) = 0$；$x = 2$。验算 $19 \times 13 = 247$ ✓。

2. **line · 第 2 —** "One factor must be 0" only works when the product IS 0. A product of 143 tells you nothing about each factor on its own.

   > 「必有一个因式为 0」只在乘积等于 0 时成立。乘积是 143，并不能单独确定每个因式。

   Expand and move: $n^2 + 2n - 143 = 0$, $(n+13)(n-11) = 0$, $n = 11$ or $n = -13$. Check $11 \times 13 = 143$ ✓.

   > 展开并移项：$n^2 + 2n - 143 = 0$，$(n+13)(n-11) = 0$，$n = 11$ 或 $n = -13$。验算 $11 \times 13 = 143$ ✓。

3. **line · 第 3 —** The habit "a word problem has one answer" kicks in. But both times are positive: the ball passes 26 m going up and again coming down.

   > 「应用题只有一个答案」的习惯在作怪。两个时间都为正：球上升时经过 26 m，下落时又经过一次。

   Keep every root that fits the story: $t = 1$ s and $t = 5$ s (the axis $t = 3$ sits halfway between them ✓).

   > 保留所有符合情境的根：$t = 1$ 秒和 $t = 5$ 秒（对称轴 $t = 3$ 正好在中间 ✓）。

4. **line · 第 1 —** "Set it equal to 0" became automatic. Zero means the ground; this question asks for a height of 51, so $h(t)$ must equal 51.

   > 「令它等于 0」成了条件反射。0 代表地面；这题问的是高 51 m，所以要令 $h(t) = 51$。

   $-5t^2 + 35t + 1 = 51 \Rightarrow -5t^2 + 35t - 50 = 0 \Rightarrow t^2 - 7t + 10 = 0 \Rightarrow (t-2)(t-5) = 0$: $t = 2$ s and $t = 5$ s.

   > $-5t^2 + 35t + 1 = 51 \Rightarrow -5t^2 + 35t - 50 = 0 \Rightarrow t^2 - 7t + 10 = 0 \Rightarrow (t-2)(t-5) = 0$：$t = 2$ 秒和 $t = 5$ 秒。

5. **line · 第 3 —** The equation does not know x is a length, so it hands back both roots. Your job is to put the story back in.

   > 方程不知道 x 是长度，所以两个根都给你。把情境放回去是你的活。

   A length cannot be -8: reject it. Legs 6 cm and 8 cm; check $36 + 64 = 100$ ✓.

   > 长度不能是 -8，舍去。两直角边 6 cm 和 8 cm；验算 $36 + 64 = 100$ ✓。