Vertex form & transformations

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Read a, p and q. Rewrite the bracket as $(x - p)$ so the sign of p is honest: $(x + 3)$ means $p = -3$.
2. 🎯 **Asked** Which one is wanted: the vertex, axis, max or min, range, the transformations, the number of x-intercepts, or the equation itself (find a)?
3. ✏️ **Picture it** Plot the vertex, draw the axis $x = p$ as a dashed line, then step out 1, 2, 3 units and go up a, $4a$, $9a$ on both sides.
4. 🧰 **Tool** To describe: a stretches or flips, p moves left or right, q moves up or down. To find the equation: write $y = a(x - p)^2 + q$ with the vertex filled in, substitute the OTHER point, solve for a.
5. ✅ **Check** Put $x = p$ into your equation: you must get q. Put the other point in: it must work. Sense check: a point below the vertex means $a < 0$.

⭐ Must know

- $y = a(x - p)^2 + q$: vertex (p, q), axis $x = p$. $(x + 3)^2$ means $p = -3$.
- $|a| > 1$ narrower, $|a| < 1$ wider, $a < 0$ flips. From the vertex: over 1, 2, 3 → up a, $4a$, $9a$.
- x-intercepts: a, q opposite signs → 2; $q = 0$ → 1; same signs → 0.

$$y = a(x - p)^2 + q$$

$$(x,\ y) \ \rightarrow \ (x + p,\ ay + q)$$

🔍 Watch out for

1. Wrong sign for p
2. Taking only the positive square root
3. Substituting the vertex to find a
4. Mixing up wider and narrower
5. Counting x-intercepts from q alone

🌱 Small wins — tick as you go

- ○ I can substitute a negative number into a squared bracket, solve $4a - 3 = 5$, and solve $(x - 1)^2 = 9$. (5′)
- ○ I can ask the five starting questions for a parabola in vertex form. (5′)
- ○ I can follow, then finish, finding a from the vertex and one other point. (8′)
- ○ I can write a vertex-form equation on my own with at most two hints. (7′)
- ○ I can spot the five classic mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Evaluate $-2(x+3)^2+5$ when $x=-1$.
A. -3 B. 13 C. 3 D. -11

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5-(-3)=5+3=8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2=16$, but $-4^2=-16$ (the square happens first).

1. $-7-(-2)$: change "minus negative" to "plus": $-7+2=-5$.
2. $(-3)(-5)=15$ (same signs), $(-3)(5)=-15$ (different signs).
3. Plugging in $x=-3$ to x^2: write brackets first, $(-3)^2=9$.

2. Solve $5=4a-3$.
A. $a=2$ B. $a=\frac{1}{2}$ C. $a=8$ D. $a=-2$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x-2)=x+8 \to 3x-6=x+8 \to 2x=14 \to x=7$.
2. $\frac{x}{2}+\frac{x}{3}=5$: multiply every term by 6 → $3x+2x=30 \to x=6$.
3. Check by substituting: $3(7-2)=15$ and $7+8=15$ ✓.

3. Solve $(x-1)^2=9$.
A. $x=4$ only B. $x=4$ or $x=-2$ C. $x=10$ or $x=-8$ D. $x=3$ or $x=-3$

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab}=\sqrt{a}\sqrt{b}$, so $\sqrt{50}=\sqrt{25}\sqrt{2}=5\sqrt{2}$. But $\sqrt{a+b}\neq\sqrt{a}+\sqrt{b}$: $\sqrt{9+16}=5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2=49$ has TWO answers $x=\pm 7$, but $\sqrt{49}=7$ only.

4. Start at $(1,2)$, move 4 units left and 3 units up. Where are you?
A. $(5,5)$ B. $(-3,5)$ C. $(-3,-1)$ D. $(5,-1)$

If you missed it — **Reading a graph: points, intercepts, slope**: A point is (x,y): across first, then up. x-intercepts are where $y=0$ (the graph crosses the horizontal axis); the y-intercept is where $x=0$. Slope $=\frac{\text{rise}}{\text{run}}=\frac{y_2-y_1}{x_2-x_1}$.

1. For $y=x^2-4$: y-intercept at $x=0$ is -4; x-intercepts solve $x^2-4=0$: $x=\pm 2$.
2. Above the x-axis means $y>0$; below means $y<0$.

Answers: 1: A · 2: A · 3: B · 4: B

🔍 Common-errors sheet

1. Wrong sign for p

State the vertex of $y = 3(x + 2)^2 - 7$.

1 $p = 2$ and $q = -7$.

2 Vertex $(2, -7)$.

✍ Which line goes wrong first, and why?

2. Taking only the positive square root

Find the x-intercepts of $y = 2(x - 3)^2 - 8$.

1 Set $y = 0$: $2(x - 3)^2 = 8$.

2 $(x - 3)^2 = 4$.

3 $x - 3 = 2$, so $x = 5$.

✍ Which line goes wrong first, and why?

3. Substituting the vertex to find a

Find the equation of the parabola with vertex $(3, 2)$ through $(5, 10)$.

1 $y = a(x - 3)^2 + 2$.

2 Substitute $(3, 2)$: $2 = a(0)^2 + 2$.

3 $2 = 2$ works for any a, so take $a = 1$: $y = (x - 3)^2 + 2$.

✍ Which line goes wrong first, and why?

4. Mixing up wider and narrower

Describe how $y = \frac{1}{3}(x-1)^2 + 4$ is obtained from $y = x^2$.

1 $a = \frac{1}{3}$, so the graph is stretched by a factor of 3 and gets narrower.

2 Then it moves 1 right and 4 up.

✍ Which line goes wrong first, and why?

5. Counting x-intercepts from q alone

How many x-intercepts does $y = -3(x+1)^2 - 2$ have?

1 $q = -2$, so the vertex $(-1, -2)$ is below the x-axis.

2 A vertex below the axis means the graph crosses it twice: 2 intercepts.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1 —** The form is $(x - p)$, so $(x + 2) = (x - (-2))$ means $p = -2$. The bracket is zero at $x = -2$, and that is where the vertex sits. $p = -2$, $q = -7$: vertex $(-2, -7)$.
2. **line 3 —** Both 2^2 and $(-2)^2$ equal 4. A parabola is symmetric, so an intercept at 5 (2 right of the axis $x = 3$) has a partner 2 to the left. $x - 3 = \pm 2$, so $x = 5$ or $x = 1$.
3. **line 2 —** The vertex is already built into the equation, so it always fits and tells you nothing about a. Only the other point pins a down. Use $(5, 10)$: $10 = a(2)^2 + 2 = 4a + 2$, so $a = 2$ and $y = 2(x-3)^2 + 2$.
4. **line 1 —** The factor is a itself, not its flip. Every height is multiplied by $\frac{1}{3}$: from the vertex, over 3 goes up only $9 \cdot \frac{1}{3} = 3$ instead of 9, so the graph is flatter and wider. Vertical stretch by a factor of $\frac{1}{3}$ (wider), then 1 right and 4 up. Vertex $(1, 4)$.
5. **line 2 —** That is only true if the parabola opens UP. Here $a = -3 < 0$, so it opens down from a vertex that is already below the axis and never reaches it. a and q are both negative (same signs), so there are 0 x-intercepts. Check: $-3(x+1)^2 = 2$ needs a square equal to $-\frac{2}{3}$, which is impossible.