Standard form → vertex form

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Write a, b, c with their signs. Note whether a is 1, another positive number, or negative.
2. 🎯 **Asked** The form $y = a(x - p)^2 + q$ with the SAME a, plus the vertex (p, q) and, if asked, the max/min value or the range.
3. ✏️ **Picture it** Sketch where the vertex should be: $p = -\frac{b}{2a}$ sets the axis, and the sign of a says whether q is a minimum or a maximum.
4. 🧰 **Tool** Factor a out of the x^2 and x terms only. Inside, add and subtract $\left(\frac{b'}{2}\right)^2$, where b' is the new x-coefficient. Move the subtracted number out, multiplied by a.
5. ✅ **Check** Your p must equal $-\frac{b}{2a}$, and $f(p)$ in the ORIGINAL must equal your q. Expanding back is the full check.

⭐ Must know

- $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
- $a \neq 1$: factor a from the x^2 and x terms only; the subtracted square leaves the bracket times a.
- Check: vertex $x = -\frac{b}{2a}$, vertex $y = f\left(-\frac{b}{2a}\right)$.

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$y = ax^2 + bx + c \;\Rightarrow\; p = -\frac{b}{2a},\; q = f(p)$$

🔍 Watch out for

1. Moving the number out without multiplying by a
2. Wrong sign inside the bracket
3. Adding the square but never subtracting it
4. Factoring out a negative a but keeping the old sign
5. Using b instead of half of b

🌱 Small wins — tick as you go

- ○ I can expand $(x - 4)^2$, spot a perfect square like $x^2 + 10x + 25$, square a fraction, and distribute -2. (5′)
- ○ I can say what vertex form will look like before I start, and work out $p = -\frac{b}{2a}$ as my target. (5′)
- ○ I can follow, then finish, the factor–add/subtract–move-out method when $a \neq 1$. (9′)
- ○ I can convert a quadratic with negative a on my own, with at most two hints, and give the maximum. (8′)
- ○ I can spot the five classic completing-the-square mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions and check each vertex with $-\frac{b}{2a}$. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Expand $(x-4)^2$.
A. x^2-16 B. x^2+16 C. $x^2-8x+16$ D. $x^2-4x+16$

If you missed it — **Multiplying binomials,** $(a+b)^2$: Every term times every term (FOIL): $(x+3)(x-5)=x^2-5x+3x-15=x^2-2x-15$. A square has a middle term: $(x+3)^2=x^2+6x+9$, never x^2+9.

1. $(x-4)^2=(x-4)(x-4)=x^2-4x-4x+16=x^2-8x+16$.
2. Shortcut: $(a\pm b)^2=a^2\pm 2ab+b^2$ — middle term $=2\times$ first $\times$ last.

2. Which perfect square equals $x^2+10x+25$?
A. $(x+25)^2$ B. $(x+5)^2$ C. $(x+10)^2$ D. $(x-5)^2$

If you missed it — **Factoring x^2+bx+c (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: x^2-x-12 → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2+7x+10$: $2\times 5=10$, $2+5=7$ → $(x+2)(x+5)$.
2. If $c<0$ the two numbers have different signs; if $c>0$ they share the sign of b.

3. What is $\left(-\frac{3}{2}\right)^2$?
A. $-\frac{9}{4}$ B. -3 C. $\frac{9}{2}$ D. $\frac{9}{4}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3}-\frac{1}{4}$: common denominator 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$.
2. $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$.
3. Never add denominators: $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$.

4. Expand $-2(x^2-4x)$.
A. $-2x^2+8x$ B. $-2x^2-8x$ C. $-2x^2-4x$ D. $2x^2-8x$

If you missed it — **Distributing (and the minus sign)**: Multiply EVERY term inside the bracket: $3(x-4)=3x-12$. A minus in front changes every sign inside: $-(x-5)=-x+5$, and $2-(x+3)=2-x-3=-x-1$.

1. $-2(3x-1)=-6x+2$ (the -1 becomes $+2$).
2. $x(x+2)-3(x+2)=x^2+2x-3x-6=x^2-x-6$.

Answers: 1: C · 2: B · 3: D · 4: A

🔍 Common-errors sheet

1. Moving the number out without multiplying by a

Write $y = 2x^2 + 12x + 5$ in vertex form.

1 $y = 2(x^2 + 6x + 9 - 9) + 5$

2 $y = 2(x + 3)^2 - 9 + 5$

3 $y = 2(x + 3)^2 - 4$

✍ Which line goes wrong first, and why?

2. Wrong sign inside the bracket

Write $y = x^2 + 8x + 3$ in vertex form and give the vertex.

1 Half of 8 is 4: $y = (x - 4)^2 - 16 + 3$

2 $y = (x - 4)^2 - 13$, vertex $(4, -13)$

✍ Which line goes wrong first, and why?

3. Adding the square but never subtracting it

Write $y = x^2 - 6x + 2$ in vertex form.

1 Complete the square: $y = (x^2 - 6x + 9) + 2$

2 $y = (x - 3)^2 + 2$

✍ Which line goes wrong first, and why?

4. Factoring out a negative a but keeping the old sign

Write $y = -x^2 + 6x + 1$ in vertex form.

1 $y = -(x^2 + 6x) + 1$

2 $y = -(x^2 + 6x + 9 - 9) + 1 = -(x + 3)^2 + 10$

✍ Which line goes wrong first, and why?

5. Using b instead of half of b

Write $y = x^2 + 4x - 1$ in vertex form.

1 $y = (x + 4)^2 - 16 - 1$

2 $y = (x + 4)^2 - 17$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** The -9 was inside a bracket that is multiplied by 2, so it is really worth -18. Lifting it out "as is" changes the value. $2(x + 3)^2 - 18 + 5 = 2(x + 3)^2 - 13$. Check: $f(-3) = 18 - 36 + 5 = -13$ ✓.
2. **line 1 —** Vertex form has a minus built in, $(x - p)^2$, so it feels like the bracket "should" have a minus. But $(x - 4)^2$ expands to $x^2 - 8x + \ldots$, not $+8x$. $y = (x + 4)^2 - 16 + 3 = (x + 4)^2 - 13$, vertex $(-4, -13)$. Check: $-\frac{8}{2} = -4$ ✓.
3. **line 1 —** Adding 9 is fine for an equation if you add it to both sides, but here there is only one side. Adding 9 alone makes a different parabola. Add AND subtract 9: $y = (x^2 - 6x + 9) - 9 + 2 = (x - 3)^2 - 7$. Check: $f(3) = 9 - 18 + 2 = -7$ ✓.
4. **line 1 —** Taking out -1 divides EVERY term in the bracket by -1, so $+6x$ becomes $-6x$. Only the x^2 term was changed. $y = -(x^2 - 6x) + 1 = -(x^2 - 6x + 9 - 9) + 1 = -(x - 3)^2 + 9 + 1 = -(x - 3)^2 + 10$. Check: $p = -\frac{6}{2(-1)} = 3$ ✓.
5. **line 1 —** The middle term of $(x + k)^2$ is $2kx$, so k is HALF of the x-coefficient. $(x + 4)^2$ would give $8x$. Half of 4 is 2: $y = (x + 2)^2 - 4 - 1 = (x + 2)^2 - 5$. Check: $f(-2) = 4 - 8 - 1 = -5$ ✓.