Max/min problems

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** What changes, and by how much each step (price step, sales lost, metres of fence)? Name the variable and say what it counts, e.g. "n = number of 1-dollar increases".
2. 🎯 **Asked** Is it asking WHEN or WHICH value (the x-coordinate of the vertex), HOW MUCH (the y-coordinate), or both? In what units?
3. ✏️ **Picture it** Sketch the shape (a pen against a wall, three widths) or make a small table of n, price and number sold. The graph is a downward parabola; the answer is at the top.
4. 🧰 **Tool** Write the quantity as a product of two linear factors, e.g. (price)(number sold) or (width)(length). Find where each factor is zero; the vertex is halfway between. If it is already expanded, use $-\dfrac{b}{2a}$.
5. ✅ **Check** Try one step either side of your answer: both must give less. Then answer the question asked in a sentence with units, and make sure it fits the story (price and sides positive).

⭐ Must know

- Optimize = find the vertex. "When / which value" → x-coordinate; "how much" → y-coordinate.
- Build the function first: area $= lw$, revenue $=$ price $\times$ number sold.
- Product of two linear factors: best x = midpoint of their zeros.

$$x = -\frac{b}{2a}$$

$$R = (\text{price}) \times (\text{quantity})$$

🔍 Watch out for

1. Answering with the wrong coordinate
2. Giving n as the price
3. Fencing the side that has a wall
4. Dropping the 2 in the vertex formula
5. Losing 1 sale per step instead of the given drop

🌱 Small wins — tick as you go

- ○ I can expand $(5+n)(40-2n)$, compute $-\dfrac{b}{2a}$ with negatives, and find the zeros of a product. (5′)
- ○ I can ask the five starting questions for a max/min word problem. (5′)
- ○ I can follow, then finish, a revenue problem: name n, write (price)(number sold), take the midpoint of the zeros. (9′)
- ○ I can solve a revenue problem on my own with at most two hints and answer in a sentence with units. (9′)
- ○ I can spot the five classic mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Expand $(5+n)(40-2n)$.
A. $-2n^2+30n+200$ B. $-2n^2+50n+200$ C. $2n^2+30n+200$ D. $-2n^2+30n+45$

If you missed it — **Distributing (and the minus sign)**: Multiply EVERY term inside the bracket: $3(x-4)=3x-12$. A minus in front changes every sign inside: $-(x-5)=-x+5$, and $2-(x+3)=2-x-3=-x-1$.

1. $-2(3x-1)=-6x+2$ (the -1 becomes $+2$).
2. $x(x+2)-3(x+2)=x^2+2x-3x-6=x^2-x-6$.

2. Find $-\dfrac{b}{2a}$ when $a=-4$ and $b=24$.
A. 3 B. -3 C. 6 D. -6

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5-(-3)=5+3=8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2=16$, but $-4^2=-16$ (the square happens first).

1. $-7-(-2)$: change "minus negative" to "plus": $-7+2=-5$.
2. $(-3)(-5)=15$ (same signs), $(-3)(5)=-15$ (different signs).
3. Plugging in $x=-3$ to x^2: write brackets first, $(-3)^2=9$.

3. For which values of x is $(x+6)(18-3x)=0$?
A. $x=-6$ or $x=6$ B. $x=6$ or $x=-18$ C. $x=-6$ or $x=18$ D. $x=6$ or $x=3$

If you missed it — **Factoring x^2+bx+c (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: x^2-x-12 → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2+7x+10$: $2\times5=10$, $2+5=7$ → $(x+2)(x+5)$.
2. If $c<0$ the two numbers have different signs; if $c>0$ they share the sign of b.

4. A fence satisfies $2w+l=100$. Write l in terms of w.
A. $l=100-2w$ B. $l=50-w$ C. $l=100+2w$ D. $l=\dfrac{100-w}{2}$

If you missed it — **Rearranging a formula**: Treat the other letters as numbers and undo operations in reverse order. $A=P(1+r)$ for r: divide by P, then subtract 1: $r=\frac{A}{P}-1$.

1. $y=a(x-p)^2+q$ for a: subtract q, divide by $(x-p)^2$: $a=\frac{y-q}{(x-p)^2}$.
2. Substitute numbers first if the question gives them — fewer letters, fewer slips.

Answers: 1: A · 2: A · 3: A · 4: A

🔍 Common-errors sheet

1. Answering with the wrong coordinate

A ball's height is $h(t) = -5t^2 + 20t + 1$ (metres, t in seconds). What is its maximum height?

1 $t = -\dfrac{20}{2(-5)} = 2.$

2 The maximum height is 2.

✍ Which line goes wrong first, and why?

2. Giving n as the price

A club sells 150 shirts at 12 dollars each. Each 1-dollar increase loses 5 sales. What price maximizes revenue?

1 $R = (12 + n)(150 - 5n)$; zeros $n = -12$ and $n = 30$.

2 Midpoint: $n = \dfrac{-12 + 30}{2} = 9.$

3 The best price is 9 dollars.

✍ Which line goes wrong first, and why?

3. Fencing the side that has a wall

100 m of fence makes a rectangular pen against a wall, fenced on three sides only. What is the largest area?

1 Perimeter: $2l + 2w = 100$, so $l = 50 - w$.

2 $A = w(50 - w)$; zeros 0 and 50, so $w = 25$ and $A = 625$ m².

✍ Which line goes wrong first, and why?

4. Dropping the 2 in the vertex formula

A ball's height is $h(t) = -4t^2 + 24t + 5$ (metres). Find the maximum height.

1 $t = -\dfrac{24}{-4} = 6.$

2 $h(6) = -144 + 144 + 5 = 5$ m.

✍ Which line goes wrong first, and why?

5. Losing 1 sale per step instead of the given drop

A museum has 400 visitors a day at 14 dollars each. Each 1-dollar increase loses 20 visitors. What price maximizes revenue?

1 Let n = number of 1-dollar increases. Price $14 + n$, visitors $400 - n$.

2 $R = (14 + n)(400 - n)$; zeros -14 and 400, so $n = 193$.

3 Price $14 + 193 = 207$ dollars.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** $t = 2$ tells you WHEN the top happens. The question asks HOW HIGH, which is the h-value at that time. $h(2) = -20 + 40 + 1 = 21$. The maximum height is 21 m, reached after 2 s.
2. **line 3 —** n counts the 1-dollar INCREASES, not the price. A price of 9 dollars would be a price cut, which the model never described. Price $= 12 + 9 = 21$ dollars, sales $150 - 45 = 105$, revenue $21 \times 105 = 2205$ dollars.
3. **line 1 —** The wall is free; only two widths and ONE length use fence. Using the full perimeter spends fence on a side that needs none. $2w + l = 100$, $l = 100 - 2w$, $A = w(100 - 2w)$; zeros 0 and 50, $w = 25$, $l = 50$, $A = 1250$ m².
4. **line 1 —** The formula is $-\dfrac{b}{2a}$. Without the 2 you land at the time the ball comes back to its starting height, which is the clue: the "maximum" equals the starting height 5. $t = -\dfrac{24}{2(-4)} = 3$; $h(3) = -36 + 72 + 5 = 41$ m.
5. **line 1 —** Each increase loses 20 visitors, so after n increases $20n$ are gone. Writing $400 - n$ means only one visitor leaves per step, and the answer (207 dollars for a museum) fails the sense check. Visitors $400 - 20n$. $R = (14 + n)(400 - 20n)$, zeros -14 and 20, $n = 3$: price 17 dollars, 340 visitors, $R = 5780$ dollars.