Factored form & writing equations

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Write the zeros (x-intercepts) and the one extra point. A graph that only touches the x-axis gives a double zero.
2. 🎯 **Asked** Which form is wanted: factored, vertex or standard? Or is it the vertex or the max/min value?
3. ✏️ **Picture it** Mark the zeros on the x-axis and the axis halfway between them. If the extra point is above the x-axis between the zeros, the parabola opens down, so expect $a < 0$.
4. 🧰 **Tool** Write $y = a(x - r)(x - s)$ with the bracket signs opposite to the zeros, put in the extra point and solve for a.
5. ✅ **Check** The extra point must fit, and the sign of a must match your sketch. Expand only at the end if standard form is asked.

⭐ Must know

- $y = a(x - r)(x - s)$: zeros r, s (opposite signs to the brackets); axis $x = \frac{r+s}{2}$.
- Zeros alone do not fix the parabola: always use a third point to find a.
- Touches the x-axis only at r: $y = a(x - r)^2$.

$$y = a(x - r)(x - s)$$

$$x = \frac{r + s}{2}$$

🔍 Watch out for

1. Reading the zeros with the bracket signs
2. Leaving out a
3. Dropping a when finding the vertex height
4. Losing the sign when solving for a
5. Dropping the common factor

🌱 Small wins — tick as you go

- ○ I can solve $(x + 4)(x - 3) = 0$, solve $12 = -6a$, and expand $-(x - 2)(x - 6)$. (5′)
- ○ I can ask the five starting questions when a question gives zeros and one more point. (5′)
- ○ I can follow, then finish, a zeros-then-a solution and find the vertex from it. (8′)
- ○ I can write a factored-form equation from the zeros and a maximum value with at most two hints. (7′)
- ○ I can spot the five classic factored-form mistakes. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

🧰 Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Solve $(x+4)(x-3)=0$.
A. $x=4$ or $x=-3$ B. $x=-4$ or $x=3$ C. $x=4$ or $x=3$ D. $x=-12$

✎ If you missed it — **Factoring x^2+bx+c (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: x^2-x-12 → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2+7x+10$: $2\times5=10$, $2+5=7$ → $(x+2)(x+5)$.
2. If $c<0$ the two numbers have different signs; if $c>0$ they share the sign of b.

2. Solve $12=-6a$.
A. $a=2$ B. $a=-2$ C. $a=18$ D. $a=-\frac{1}{2}$

✎ If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x-2)=x+8$ → $3x-6=x+8$ → $2x=14$ → $x=7$.
2. $\frac{x}{2}+\frac{x}{3}=5$: multiply every term by 6 → $3x+2x=30$ → $x=6$.
3. Check by substituting: $3(7-2)=15$ and $7+8=15$ ✓.

3. Evaluate $(1+2)(1-4)$.
A. 9 B. -9 C. -3 D. 3

✎ If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5-(-3)=5+3=8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2=16$, but $-4^2=-16$ (the square happens first).

1. $-7-(-2)$: change "minus negative" to "plus": $-7+2=-5$.
2. $(-3)(-5)=15$ (same signs), $(-3)(5)=-15$ (different signs).
3. Plugging in $x=-3$ to x^2: write brackets first, $(-3)^2=9$.

4. What number is halfway between -3 and 7?
A. 5 B. 2 C. 4 D. -5

✎ If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3}-\frac{1}{4}$: common denominator 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$.
2. $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$.
3. Never add denominators: $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$.

5. Expand $-(x-2)(x-6)$.
A. $-x^2+8x-12$ B. $-x^2-8x-12$ C. $x^2-8x+12$ D. $-x^2+8x+12$

✎ If you missed it — **Distributing (and the minus sign)**: Multiply EVERY term inside the bracket: $3(x-4)=3x-12$. A minus in front changes every sign inside: $-(x-5)=-x+5$, and $2-(x+3)=2-x-3=-x-1$.

1. $-2(3x-1)=-6x+2$ (the -1 becomes $+2$).
2. $x(x+2)-3(x+2)=x^2+2x-3x-6=x^2-x-6$.

Answers: 1: B · 2: B · 3: B · 4: B · 5: A

🔍 Common-errors sheet

1. Reading the zeros with the bracket signs

Find the x-intercepts and the axis of $y = 3(x + 2)(x - 8)$.

1 x-intercepts: $x = 2$ and $x = -8$

2 Axis: $x = \dfrac{2 + (-8)}{2} = -3$

✍ Which line goes wrong first, and why?

2. Leaving out a

A parabola has x-intercepts -1 and 3 and passes through $(1, -8)$. Find its equation.

1 $y = (x + 1)(x - 3)$

2 Expanded: $y = x^2 - 2x - 3$

✍ Which line goes wrong first, and why?

3. Dropping a when finding the vertex height

Find the vertex of $y = -3(x - 1)(x - 7)$.

1 Axis: $x = \dfrac{1 + 7}{2} = 4$

2 $y = (4 - 1)(4 - 7) = -9$

3 Vertex $(4, -9)$

✍ Which line goes wrong first, and why?

4. Losing the sign when solving for a

A parabola has x-intercepts 1 and 7 and passes through $(5, 16)$. Find its equation.

1 $y = a(x - 1)(x - 7);\ 16 = a(4)(-2)$

2 $a = \dfrac{16}{8} = 2$

3 $y = 2(x - 1)(x - 7)$

✍ Which line goes wrong first, and why?

5. Dropping the common factor

Write $y = 3x^2 - 6x - 24$ in factored form.

1 $3x^2 - 6x - 24 = 3(x^2 - 2x - 8)$

2 $y = (x - 4)(x + 2)$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1 —** The numbers were copied straight out of the brackets. A zero is where a bracket equals 0: $x + 2 = 0$ gives $x = -2$. Zeros -2 and 8; axis $x = \dfrac{-2 + 8}{2} = 3$.
2. **line 1 —** Many parabolas share the same two zeros; they differ only by a. The point $(1, -8)$ was never used, and at $x = 1$ this gives -4, not -8. $y = a(x + 1)(x - 3)$; $-8 = a(2)(-2) = -4a$, so $a = 2$.
3. **line 2 —** The brackets were worked out, and the -3 in front was forgotten. It multiplies everything. $y = -3(3)(-3) = 27$. Opens down, so $(4, 27)$ is a maximum.
4. **line 2 —** $(4)(-2)$ is -8, not 8. A quick picture warns you too: a point above the x-axis between the zeros means the parabola opens down. $16 = -8a$, so $a = -2$.
5. **line 2 —** The 3 was used to make the trinomial easier, then left behind. Without it the curve is only a third as steep, and every y-value is wrong. Keep the 3 in front: $y = 3(x - 4)(x + 2)$.