Factored form & writing equations

交点式与求解析式

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Write the zeros (x-intercepts) and the one extra point. A graph that only touches the x-axis gives a double zero.
   > 写下零点（x 截距）和多给的那个点。图象只碰到 x 轴的，是一个重零点。
2. 🎯 **Asked · 求什么** Which form is wanted: factored, vertex or standard? Or is it the vertex or the max/min value?
   > 题目要哪种形式：交点式、顶点式还是一般式？还是要顶点或最值？
3. ✏️ **Picture it · 画出来** Mark the zeros on the x-axis and the axis halfway between them. If the extra point is above the x-axis between the zeros, the parabola opens down, so expect $a < 0$.
   > 在 x 轴上标出零点，再在中间画对称轴。如果多给的点在两个零点之间且在 x 轴上方，抛物线开口向下，a 应为负。
4. 🧰 **Tool · 用什么工具** Write $y = a(x - r)(x - s)$ with the bracket signs opposite to the zeros, put in the extra point and solve for a.
   > 写出 $y = a(x - r)(x - s)$，括号里的符号与零点相反；代入多给的点，解出 a。
5. ✅ **Check · 检验** The extra point must fit, and the sign of a must match your sketch. Expand only at the end if standard form is asked.
   > 多给的点必须满足方程，a 的符号要和草图一致。题目要一般式时，最后再展开。

⭐ Must know

必须记住

- $y = a(x - r)(x - s)$: zeros r, s (opposite signs to the brackets); axis $x = \frac{r+s}{2}$.
  > $y = a(x - r)(x - s)$：零点 r, s（与括号里符号相反）；对称轴 $x = \frac{r+s}{2}$。
- Zeros alone do not fix the parabola: always use a third point to find a.
  > 只有零点定不出抛物线：一定要用第三个点求 a。
- Touches the x-axis only at r: $y = a(x - r)^2$.
  > 只在 r 处与 x 轴相切：$y = a(x - r)^2$。

$$y = a(x - r)(x - s)$$

$$x = \frac{r + s}{2}$$

🔍 Watch out for

当心

1. Reading the zeros with the bracket signs
   > 照括号里的符号读零点
2. Leaving out a
   > 漏掉了 a
3. Dropping a when finding the vertex height
   > 求顶点高度时丢了 a
4. Losing the sign when solving for a
   > 解 a 时丢了符号
5. Dropping the common factor
   > 丢了公因数

🌱 Small wins — tick as you go

一小步一小步——做完打勾

○ I can solve $(x+4)(x-3)=0$, solve $12=-6a$, and expand $-(x-2)(x-6)$.

> 我会解 $(x+4)(x-3)=0$，会解 $12=-6a$，也会展开 $-(x-2)(x-6)$。

(5′)

○ I can ask the five starting questions when a question gives zeros and one more point.

> 题目给出零点和另一个点时，我能问出五个入手问题。

(5′)

○ I can follow, then finish, a zeros-then-a solution and find the vertex from it.

> 我能看懂并补全「先写零点、再求 a」的解法，并由它求出顶点。

(8′)

○ I can write a factored-form equation from the zeros and a maximum value with at most two hints.

> 我能在最多两个提示下，由零点和最大值写出交点式方程。

(7′)

○ I can spot the five classic factored-form mistakes.

> 我能找出交点式的五个经典错误。

(8′)

○ I can do the Level 1–2 practice questions.

> 我能做出 1–2 级练习题。

(10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Solve $(x+4)(x-3)=0$.

> 解 $(x+4)(x-3)=0$。

A. $x=4$ or $x=-3$ B. $x=-4$ or $x=3$ C. $x=4$ or $x=3$ D. $x=-12$

If you missed it · 如果做错了 — **Factoring x^2+bx+c (product and sum)**

> **分解 x^2+bx+c（积与和）**

: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: x^2-x-12 → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

> 先提公因式。再找两个数，相乘等于 c、相加等于 b：x^2-x-12 → -4 和 3 → $(x-4)(x+3)$。展开检验。

1. $x^2+7x+10$: $2\times5=10$, $2+5=7$ → $(x+2)(x+5)$.
   > $x^2+7x+10$：$2\times5=10$，$2+5=7$ → $(x+2)(x+5)$。
2. If $c<0$ the two numbers have different signs; if $c>0$ they share the sign of b.
   > 若 $c<0$，两个数异号；若 $c>0$，两个数与 b 同号。

2. Solve $12=-6a$.

> 解 $12=-6a$。

A. $a=2$ B. $a=-2$ C. $a=18$ D. $a=-\frac{1}{2}$

If you missed it · 如果做错了 — **Solving a linear equation**

> **解一元一次方程**

: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

> 两边做同样的事。先去括号，把含 x 的项移到一边、数字移到另一边，最后除以系数。有分数时先把每一项都乘以公分母。

1. $3(x-2)=x+8$ → $3x-6=x+8$ → $2x=14$ → $x=7$.
   > $3(x-2)=x+8$ → $3x-6=x+8$ → $2x=14$ → $x=7$。
2. $\frac{x}{2}+\frac{x}{3}=5$: multiply every term by 6 → $3x+2x=30$ → $x=6$.
   > $\frac{x}{2}+\frac{x}{3}=5$：每项乘 6 → $3x+2x=30$ → $x=6$。
3. Check by substituting: $3(7-2)=15$ and $7+8=15$ ✓.
   > 代回检验：$3(7-2)=15$，$7+8=15$ ✓。

3. Evaluate $(1+2)(1-4)$.

> 求 $(1+2)(1-4)$ 的值。

A. 9 B. -9 C. -3 D. 3

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5-(-3)=5+3=8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2=16$, but $-4^2=-16$ (the square happens first).

> 减去负数等于加上它：$5-(-3)=5+3=8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2=16$，但 $-4^2=-16$（先算平方）。

1. $-7-(-2)$: change "minus negative" to "plus": $-7+2=-5$.
   > $-7-(-2)$：把「减负」改成「加」：$-7+2=-5$。
2. $(-3)(-5)=15$ (same signs), $(-3)(5)=-15$ (different signs).
   > $(-3)(-5)=15$（同号），$(-3)(5)=-15$（异号）。
3. Plugging in $x=-3$ to x^2: write brackets first, $(-3)^2=9$.
   > 把 $x=-3$ 代入 x^2：先加括号，$(-3)^2=9$。

4. What number is halfway between -3 and 7?

> 正好在 -3 和 7 中间的数是多少?

A. 5 B. 2 C. 4 D. -5

If you missed it · 如果做错了 — **Fractions: add, subtract, multiply, divide**

> **分数的四则运算**

: Add or subtract only with a common denominator: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

> 加减必须先通分：$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$。乘法分子乘分子、分母乘分母。除以一个分数＝乘它的倒数（把第二个分数倒过来）。

1. $\frac{2}{3}-\frac{1}{4}$: common denominator 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$.
   > $\frac{2}{3}-\frac{1}{4}$：公分母 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$。
2. $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$.
   > $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$。
3. Never add denominators: $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$.
   > 分母不能相加：$\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$。

5. Expand $-(x-2)(x-6)$.

> 展开 $-(x-2)(x-6)$。

A. $-x^2+8x-12$ B. $-x^2-8x-12$ C. $x^2-8x+12$ D. $-x^2+8x+12$

If you missed it · 如果做错了 — **Distributing (and the minus sign)**

> **去括号（尤其是负号）**

: Multiply EVERY term inside the bracket: $3(x-4)=3x-12$. A minus in front changes every sign inside: $-(x-5)=-x+5$, and $2-(x+3)=2-x-3=-x-1$.

> 括号里的每一项都要乘到：$3(x-4)=3x-12$。前面是负号时，括号里每一项都变号：$-(x-5)=-x+5$，$2-(x+3)=2-x-3=-x-1$。

1. $-2(3x-1)=-6x+2$ (the -1 becomes $+2$).
   > $-2(3x-1)=-6x+2$（-1 变成 $+2$）。
2. $x(x+2)-3(x+2)=x^2+2x-3x-6=x^2-x-6$.
   > $x(x+2)-3(x+2)=x^2+2x-3x-6=x^2-x-6$。

Answers · 答案: 1: B · 2: B · 3: B · 4: B · 5: A

🔍 Common-errors sheet

常见错误单

1. Reading the zeros with the bracket signs

> **照括号里的符号读零点**

Find the x-intercepts and the axis of $y = 3(x + 2)(x - 8)$.

> 求 $y = 3(x + 2)(x - 8)$ 的 x 截距和对称轴。

1 x-intercepts: $x = 2$ and $x = -8$

> x 截距：$x = 2$ 和 $x = -8$

2 Axis: $x = \dfrac{2 + (-8)}{2} = -3$

> 对称轴：$x = \dfrac{2 + (-8)}{2} = -3$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Leaving out a

> **漏掉了 a**

A parabola has x-intercepts -1 and 3 and passes through $(1, -8)$. Find its equation.

> 一条抛物线的 x 截距为 -1 和 3，且过点 $(1, -8)$。求它的方程。

1 $y = (x + 1)(x - 3)$

2 Expanded: $y = x^2 - 2x - 3$

> 展开：$y = x^2 - 2x - 3$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Dropping a when finding the vertex height

> **求顶点高度时丢了 a**

Find the vertex of $y = -3(x-1)(x-7)$.

> 求 $y = -3(x-1)(x-7)$ 的顶点。

1 Axis: $x = \dfrac{1+7}{2} = 4$

> 对称轴：$x = \dfrac{1+7}{2} = 4$

2 $y = (4-1)(4-7) = -9$

3 Vertex $(4, -9)$

> 顶点 $(4, -9)$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Losing the sign when solving for a

> **解 a 时丢了符号**

A parabola has x-intercepts 1 and 7 and passes through $(5, 16)$. Find its equation.

> 一条抛物线的 x 截距为 1 和 7，且过点 $(5, 16)$。求它的方程。

1 $y = a(x-1)(x-7);\ 16 = a(4)(-2)$

> $y = a(x-1)(x-7);\ 16 = a(4)(-2)$

2 $a = \dfrac{16}{8} = 2$

3 $y = 2(x-1)(x-7)$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Dropping the common factor

> **丢了公因数**

Write $y = 3x^2 - 6x - 24$ in factored form.

> 把 $y = 3x^2 - 6x - 24$ 写成交点式。

1 $3x^2 - 6x - 24 = 3(x^2 - 2x - 8)$

2 $y = (x-4)(x+2)$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 1 —** The numbers were copied straight out of the brackets. A zero is where a bracket equals 0: $x + 2 = 0$ gives $x = -2$.

   > 直接把括号里的数抄了出来。零点是让括号等于 0 的值：$x + 2 = 0$ 得 $x = -2$。

   Zeros -2 and 8; axis $x = \dfrac{-2 + 8}{2} = 3$.

   > 零点 -2 和 8；对称轴 $x = \dfrac{-2 + 8}{2} = 3$。

2. **line · 第 1 —** Many parabolas share the same two zeros; they differ only by a. The point $(1, -8)$ was never used, and at $x = 1$ this gives -4, not -8.

   > 有无数条抛物线有相同的两个零点，它们只差一个 a。点 $(1, -8)$ 根本没用上，而且这个式子在 $x = 1$ 时是 -4，不是 -8。

   $y = a(x + 1)(x - 3)$; $-8 = a(2)(-2) = -4a$, so $a = 2$.

   > $y = a(x + 1)(x - 3)$；$-8 = a(2)(-2) = -4a$，得 $a = 2$。

3. **line · 第 2 —** The brackets were worked out, and the -3 in front was forgotten. It multiplies everything.

   > 只算了两个括号，忘了前面的 -3。它要乘整个式子。

   $y = -3(3)(-3) = 27$. Opens down, so $(4, 27)$ is a maximum.

   > $y = -3(3)(-3) = 27$。开口向下，$(4, 27)$ 是最高点。

4. **line · 第 2 —** $(4)(-2)$ is -8, not 8. A quick picture warns you too: a point above the x-axis between the zeros means the parabola opens down.

   > $(4)(-2)$ 是 -8，不是 8。画个草图也能提醒你：两个零点之间、x 轴上方的点说明开口向下。

   $16 = -8a$, so $a = -2$.

   > $16 = -8a$，所以 $a = -2$。

5. **line · 第 2 —** The 3 was used to make the trinomial easier, then left behind. Without it the curve is only a third as steep, and every y-value is wrong.

   > 提出 3 是为了让三项式好分解，结果把它落下了。没有它，曲线只有原来三分之一陡，每个 y 值都不对。

   Keep the 3 in front: $y = 3(x - 4)(x + 2)$.

   > 把 3 留在前面：$y = 3(x - 4)(x + 2)$。