Graphs & characteristics

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Write down a, b and c with their signs. The sign of a already tells you: up (minimum) or down (maximum).
2. 🎯 **Asked** Which of the six facts are wanted? Remember that a max or min VALUE is a y-value, and the range is about y.
3. ✏️ **Picture it** A quick U or ∩ with a dashed vertical line for the axis. The vertex sits on that line, and the two x-intercepts are mirror images across it.
4. 🧰 **Tool** Axis first: $x = -\dfrac{b}{2a}$ (or the midpoint of the zeros). Put that x back into f to get the vertex height.
5. ✅ **Check** The zeros must be equally far from the axis. The range must use the vertex y-value, with $\geq$ for up and $\leq$ for down.

⭐ Must know

- $a > 0$ opens up (minimum), $a < 0$ opens down (maximum); the vertex sits on the axis $x = -\frac{b}{2a}$.
- Range uses the vertex y-value: $y \geq q$ (up) or $y \leq q$ (down). Domain is always all real numbers.
- Axis = midpoint of the zeros, or of any two points at the same height.

$$x = -\frac{b}{2a}$$

$$x_{\text{axis}} = \frac{r + s}{2}$$

🔍 Watch out for

1. Losing the minus sign in the axis
2. Writing the range with the x-value of the vertex
3. Range in the wrong direction
4. Squaring a negative input without brackets
5. Taking a given point as the top

🌱 Small wins — tick as you go

- ○ I can square a negative number with brackets, find the midpoint of two numbers, and factor $x^2 - 2x - 8$. (5′)
- ○ I can ask the five starting questions for a parabola in standard form. (5′)
- ○ I can follow, then finish, an axis-first solution that gives all six characteristics. (8′)
- ○ I can find the vertex and range of a parabola on my own with at most two hints. (8′)
- ○ I can spot the five classic mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Evaluate $-(-3)^2$.
A. 9 B. -9 C. 6 D. -6

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5-(-3)=5+3=8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2=16$, but $-4^2=-16$ (the square happens first).

1. $-7-(-2)$: change "minus negative" to "plus": $-7+2=-5$.
2. $(-3)(-5)=15$ (same signs), $(-3)(5)=-15$ (different signs).
3. Plugging in $x=-3$ to x^2: write brackets first, $(-3)^2=9$.

2. Evaluate $-\dfrac{-8}{2(2)}$.
A. 2 B. -2 C. 4 D. -4

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3}-\frac{1}{4}$: common denominator 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$.
2. $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$.
3. Never add denominators: $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$.

3. What number is exactly halfway between -5 and 1?
A. -3 B. -2 C. 2 D. 3

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4. Solve $x^2-2x-8=0$.
A. $x=-4$ or $x=2$ B. $x=4$ or $x=-2$ C. $x=8$ or $x=-1$ D. $x=2$ or $x=4$

If you missed it — **Factoring x^2+bx+c (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: x^2-x-12 → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2+7x+10$: $2\times5=10$, $2+5=7$ → $(x+2)(x+5)$.
2. If $c<0$ the two numbers have different signs; if $c>0$ they share the sign of b.

Answers: 1: B · 2: A · 3: B · 4: B

🔍 Common-errors sheet

1. Losing the minus sign in the axis

Find the vertex of $y = x^2 + 8x + 7$.

1 Axis: $x = \dfrac{b}{2a} = \dfrac{8}{2} = 4$

2 $y = 16 + 32 + 7 = 55$

3 Vertex $(4, 55)$

✍ Which line goes wrong first, and why?

2. Writing the range with the x-value of the vertex

Find the range of $y = x^2 - 10x + 21$.

1 Axis: $x = -\dfrac{-10}{2} = 5$

2 $y = 25 - 50 + 21 = -4$, vertex $(5, -4)$

3 Opens up, so the range is $y \geq 5$.

✍ Which line goes wrong first, and why?

3. Range in the wrong direction

Find the range of $f(x) = -3x^2 + 6x + 1$.

1 Axis: $x = -\dfrac{6}{2(-3)} = 1$

2 $f(1) = -3 + 6 + 1 = 4$

3 Range: $y \geq 4$

✍ Which line goes wrong first, and why?

4. Squaring a negative input without brackets

For $f(x) = -x^2 + 4x + 3$, find $f(-2)$.

1 $f(-2) = -(-2)^2 + 4(-2) + 3$

2 $= 4 - 8 + 3$

3 $= -1$

✍ Which line goes wrong first, and why?

5. Taking a given point as the top

A parabola opens down, has x-intercepts 0 and 8, and passes through $(2, 6)$. Find its maximum value.

1 $y = ax(x - 8)$; from $(2, 6)$: $6 = a(2)(-6)$, so $a = -\frac{1}{2}$

2 The highest point we know is $(2, 6)$, so the maximum is 6.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1 —** The formula is $x = -\dfrac{b}{2a}$. With b positive the axis is on the negative side. $x = -\dfrac{8}{2} = -4$, then $y = 16 - 32 + 7 = -9$.
2. **line 3 —** The range lists y-values, so it must use the height of the vertex, not where it sits left or right. The lowest y is -4, so the range is $y \geq -4$.
3. **line 3 —** Habit: most parabolas you have drawn open up. Here $a = -3 < 0$, so the vertex is the TOP and every other y is below it. Opens down, so the range is $y \leq 4$.
4. **line 2 —** The two minus signs looked like they cancel. But the square acts first: $(-2)^2 = 4$, and then the minus in front makes it -4. $-4 - 8 + 3 = -9$.
5. **line 2 —** The top is always on the axis. The point $(2, 6)$ only helps find a; it is not on the axis $x = 4$. Axis $x = \dfrac{0 + 8}{2} = 4$; $y = -\frac{1}{2}(4)(-4) = 8$.