Non-permissible values & simplifying

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Copy the denominator on its own and factor it completely. Factor the numerator too.
2. 🎯 **Asked** Two things: the non-permissible values, taken from the ORIGINAL denominator, and the simplest form with those NPVs written after it.
3. ✏️ **Picture it** A number line with an open circle at each NPV: the holes the expression can never use, even after cancelling.
4. 🧰 **Tool** Set each denominator factor $\neq 0$ first. Then cancel only whole matching FACTORS; a pair like $(3 - x)$ and $(x - 3)$ cancels to -1.
5. ✅ **Check** Choose an easy allowed value such as $x = 0$ or $x = 1$. The original and your answer must give the same number.

⭐ Must know

- NPVs: values that make the DENOMINATOR zero — factor it, set each factor $\neq 0$.
- State NPVs from the ORIGINAL expression; a cancelled factor still counts.
- Cancel FACTORS, never terms; $\frac{a-b}{b-a} = -1$.

$$\frac{AC}{BC} = \frac{A}{B}, \quad B \neq 0,\ C \neq 0$$

$$\frac{a-b}{b-a} = -1, \quad a \neq b$$

🔍 Watch out for

1. Cancelling terms instead of factors
2. Taking the NPVs from the simplified form
3. Using the numerator to find NPVs
4. Treating opposite factors as equal
5. Sign slip when taking out a negative

🌱 Small wins — tick as you go

- ○ I can factor a trinomial, simplify a number fraction, and rewrite $-(x - 5)$ as $5 - x$. (5′)
- ○ I can ask the five starting questions for a rational expression. (5′)
- ○ I can follow, then finish, a "factor, list the NPVs, cancel" simplification. (8′)
- ○ I can simplify one with an opposite factor on my own with at most two hints. (8′)
- ○ I can spot the five classic mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Factor $x^2 - x - 12$.
A. $(x-4)(x+3)$ B. $(x+4)(x-3)$ C. $(x-6)(x+2)$ D. $(x-12)(x+1)$

If you missed it — **Factoring $x^2 + bx + c$ (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12$ → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x+2)(x+5)$.
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.

2. Write $\dfrac{18}{24}$ in lowest terms.
A. $\dfrac{9}{12}$ B. $\dfrac{3}{4}$ C. $\dfrac{2}{3}$ D. $\dfrac{6}{8}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.

3. Which expression equals $-(x-5)$?
A. $5 - x$ B. $x + 5$ C. $-x - 5$ D. $x - 5$

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

4. Solve $2x + 7 = 0$.
A. $x = \frac{7}{2}$ B. $x = -\frac{7}{2}$ C. $x = -\frac{2}{7}$ D. $x = -5$

If you missed it — **Solving a linear equation**: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

1. $3(x-2) = x + 8$ → $3x - 6 = x + 8$ → $2x = 14$ → $x = 7$.
2. $\frac{x}{2} + \frac{x}{3} = 5$: multiply every term by 6 → $3x + 2x = 30$ → $x = 6$.
3. Check by substituting: $3(7-2) = 15$ and $7 + 8 = 15$ ✓.

Answers: 1: A · 2: B · 3: A · 4: B

🔍 Common-errors sheet

1. Cancelling terms instead of factors

Simplify $\dfrac{x^2+4x}{x^2+3x}$ and state the NPVs.

1 Cross out the x^2 on top and bottom: $\dfrac{4x}{3x}$.

2 Cancel x: $\dfrac{4}{3}$.

✍ Which line goes wrong first, and why?

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2. Taking the NPVs from the simplified form

Simplify $\dfrac{x^2-25}{x^2+2x-15}$ and state the NPVs.

1 Factor: $\dfrac{(x-5)(x+5)}{(x+5)(x-3)}$.

2 Cancel: $\dfrac{x-5}{x-3}$.

3 NPV: $x \neq 3$.

✍ Which line goes wrong first, and why?

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3. Using the numerator to find NPVs

State the NPVs of $\dfrac{x^2-4}{x^2-6x}$.

1 Set the top to zero: $x^2-4=(x-2)(x+2)=0$.

2 NPVs: $x \neq \pm 2$.

✍ Which line goes wrong first, and why?

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4. Treating opposite factors as equal

Simplify $\dfrac{3-x}{x^2-9}$ and state the NPVs.

1 Factor: $\dfrac{3-x}{(x-3)(x+3)}$, so $x \neq \pm 3$.

2 $3-x$ and $x-3$ are the same, so they cancel: $\dfrac{1}{x+3}$.

✍ Which line goes wrong first, and why?

5. Sign slip when taking out a negative

Simplify $\dfrac{8-2x}{x^2-8x+16}$ and state the NPV.

1 Bottom: $(x-4)^2$, so $x \neq 4$.

2 Top: $8-2x = -2(x+4)$.

3 Nothing cancels: $\dfrac{-2(x+4)}{(x-4)^2}$.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1 —** The x^2 is added to something, not multiplied, so it is a term, not a factor. Only factors that MULTIPLY the whole top and the whole bottom can cancel. Factor first: $\dfrac{x(x+4)}{x(x+3)}$, NPVs $x \neq 0,\ -3$; cancel x: $\dfrac{x+4}{x+3}$. (At $x=1$: $\frac{5}{4}$, not $\frac{4}{3}$.)
2. **line 3 —** The NPV was read off the bottom AFTER $(x+5)$ was cancelled, so $x=-5$ disappeared. At $x=-5$ the original is $\frac{0}{0}$, which is undefined. Write the NPVs from the factored original bottom $(x+5)(x-3)$: $x \neq -5,\ 3$.
3. **line 1 —** A zero on top is allowed; the value of the fraction is just 0. Only division by zero is a problem, so only the bottom matters. Bottom: $x^2-6x = x(x-6)$, so $x \neq 0$ and $x \neq 6$.
4. **line 2 —** $3-x$ and $x-3$ are opposites, not equal: at $x=5$ they are -2 and 2. Opposites cancel to -1, not to 1. $3-x = -(x-3)$, so the result is $-\dfrac{1}{x+3}$, $x \neq \pm 3$. (At $x=0$: $\frac{3}{-9} = -\frac{1}{3}$ ✓.)
5. **line 2 —** Taking out -2 flips the sign of EVERY term inside: $-2(x+4) = -2x-8$, not $8-2x$. $8-2x = -2(x-4)$ (check: $-2x+8$ ✓). Cancel one $(x-4)$: $-\dfrac{2}{x-4}$, $x \neq 4$.