Non-permissible values & simplifying

不允许值与化简

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Copy the denominator on its own and factor it completely. Factor the numerator too.
   > 把分母单独抄出来，彻底分解。分子也分解。
2. 🎯 **Asked · 求什么** Two things: the non-permissible values, taken from the ORIGINAL denominator, and the simplest form with those NPVs written after it.
   > 两样东西：不允许值（从「原」分母找），以及最简式——后面要写上这些不允许值。
3. ✏️ **Picture it · 画出来** A number line with an open circle at each NPV: the holes the expression can never use, even after cancelling.
   > 在数轴上把每个不允许值画成空心圈：这些「洞」即使约分以后也永远不能用。
4. 🧰 **Tool · 用什么工具** Set each denominator factor $\neq 0$ first. Then cancel only whole matching FACTORS; a pair like $(3-x)$ and $(x-3)$ cancels to -1.
   > 先令分母的每个因式 $\neq 0$。然后只约完全相同的「因式」；像 $(3-x)$ 和 $(x-3)$ 这样的一对约成 -1。
5. ✅ **Check · 检验** Choose an easy allowed value such as $x=0$ or $x=1$. The original and your answer must give the same number.
   > 挑一个允许的简单值，比如 $x=0$ 或 $x=1$。原式和你的答案算出来必须是同一个数。

⭐ Must know

必须记住

- NPVs: values that make the DENOMINATOR zero — factor it, set each factor $\neq 0$.
  > 不允许值：使分母为零的值——把分母分解，每个因式 $\neq 0$。
- State NPVs from the ORIGINAL expression; a cancelled factor still counts.
  > 从原式写不允许值；约掉的因式照样算。
- Cancel FACTORS, never terms; $\frac{a-b}{b-a} = -1$.
  > 只能约因式，不能约项；$\frac{a-b}{b-a} = -1$。

$$\frac{AC}{BC} = \frac{A}{B}, \quad B \neq 0,\ C \neq 0$$

$$\frac{a-b}{b-a} = -1, \quad a \neq b$$

🔍 Watch out for

当心

1. Cancelling terms instead of factors
   > 约掉的是项，不是因式
2. Taking the NPVs from the simplified form
   > 从化简后的式子找不允许值
3. Using the numerator to find NPVs
   > 用分子去找不允许值
4. Treating opposite factors as equal
   > 把相反因式当成相同因式
5. Sign slip when taking out a negative
   > 提负号时符号出错

🌱 Small wins — tick as you go

一小步一小步——做完打勾

○ I can factor a trinomial, simplify a number fraction, and rewrite $-(x-5)$ as $5-x$.

> 我会分解三项式、约简数字分数，也会把 $-(x-5)$ 改写成 $5-x$。

(5′)

○ I can ask the five starting questions for a rational expression.

> 我能对一个有理式问出五个入手问题。

(5′)

○ I can follow, then finish, a "factor, list the NPVs, cancel" simplification.

> 我能看懂并补全「分解—写不允许值—约分」的化简。

(8′)

○ I can simplify one with an opposite factor on my own with at most two hints.

> 我能在最多两个提示下，独立化简一道含相反因式的题。

(8′)

○ I can spot the five classic mistakes in someone else's work.

> 我能在别人的解答里找出五个经典错误。

(8′)

○ I can do the Level 1–2 practice questions.

> 我能做出 1–2 级练习题。

(10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Factor $x^2 - x - 12$.

> 分解因式 $x^2 - x - 12$。

A. $(x-4)(x+3)$ B. $(x+4)(x-3)$ C. $(x-6)(x+2)$ D. $(x-12)(x+1)$

If you missed it · 如果做错了 — **Factoring $x^2 + bx + c$ (product and sum)**

> **分解 $x^2 + bx + c$（积与和）**

: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12$ → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

> 先提公因式。再找两个数，相乘等于 c、相加等于 b：$x^2 - x - 12$ → -4 和 3 → $(x-4)(x+3)$。展开检验。

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x+2)(x+5)$.
   > $x^2 + 7x + 10$：$2 \times 5 = 10$，$2 + 5 = 7$ → $(x+2)(x+5)$。
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.
   > 若 $c < 0$，两个数异号；若 $c > 0$，两个数与 b 同号。

2. Write $\dfrac{18}{24}$ in lowest terms.

> 把 $\dfrac{18}{24}$ 化成最简分数。

A. $\dfrac{9}{12}$ B. $\dfrac{3}{4}$ C. $\dfrac{2}{3}$ D. $\dfrac{6}{8}$

If you missed it · 如果做错了 — **Fractions: add, subtract, multiply, divide**

> **分数的四则运算**

: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

> 加减必须先通分：$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$。乘法分子乘分子、分母乘分母。除以一个分数＝乘它的倒数（把第二个分数倒过来）。

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
   > $\frac{2}{3} - \frac{1}{4}$：公分母 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$。
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
   > $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$。
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.
   > 分母不能相加：$\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$。

3. Which expression equals $-(x-5)$?

> 哪个式子等于 $-(x-5)$?

A. $5-x$ B. $x+5$ C. $-x-5$ D. $x-5$

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5-(-3)=5+3=8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2=16$, but $-4^2=-16$ (the square happens first).

> 减去负数等于加上它：$5-(-3)=5+3=8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2=16$，但 $-4^2=-16$（先算平方）。

1. $-7-(-2)$: change "minus negative" to "plus": $-7+2=-5$.
   > $-7-(-2)$：把「减负」改成「加」：$-7+2=-5$。
2. $(-3)(-5)=15$ (same signs), $(-3)(5)=-15$ (different signs).
   > $(-3)(-5)=15$（同号），$(-3)(5)=-15$（异号）。
3. Plugging in $x=-3$ to x^2: write brackets first, $(-3)^2=9$.
   > 把 $x=-3$ 代入 x^2：先加括号，$(-3)^2=9$。

4. Solve $2x+7=0$.

> 解 $2x+7=0$。

A. $x=\frac{7}{2}$ B. $x=-\frac{7}{2}$ C. $x=-\frac{2}{7}$ D. $x=-5$

If you missed it · 如果做错了 — **Solving a linear equation**

> **解一元一次方程**

: Do the same thing to both sides. Clear brackets, collect x on one side and numbers on the other, then divide. With fractions, multiply every term by the common denominator first.

> 两边做同样的事。先去括号，把含 x 的项移到一边、数字移到另一边，最后除以系数。有分数时先把每一项都乘以公分母。

1. $3(x-2)=x+8 \to 3x-6=x+8 \to 2x=14 \to x=7$.
   > $3(x-2)=x+8 \to 3x-6=x+8 \to 2x=14 \to x=7$。
2. $\frac{x}{2}+\frac{x}{3}=5$: multiply every term by 6 → $3x+2x=30 \to x=6$.
   > $\frac{x}{2}+\frac{x}{3}=5$：每项乘 6 → $3x+2x=30 \to x=6$。
3. Check by substituting: $3(7-2)=15$ and $7+8=15$ ✓.
   > 代回检验：$3(7-2)=15$，$7+8=15$ ✓。

Answers · 答案: 1: A · 2: B · 3: A · 4: B

🔍 Common-errors sheet

常见错误单

1. Cancelling terms instead of factors

> **约掉的是项，不是因式**

Simplify $\dfrac{x^2+4x}{x^2+3x}$ and state the NPVs.

> 化简 $\dfrac{x^2+4x}{x^2+3x}$，并写出不允许值。

1 Cross out the x^2 on top and bottom: $\dfrac{4x}{3x}$.

> 把上下的 x^2 划掉：$\dfrac{4x}{3x}$。

2 Cancel x: $\dfrac{4}{3}$.

> 约去 x：$\dfrac{4}{3}$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Taking the NPVs from the simplified form

> **从化简后的式子找不允许值**

Simplify $\dfrac{x^2-25}{x^2+2x-15}$ and state the NPVs.

> 化简 $\dfrac{x^2-25}{x^2+2x-15}$，并写出不允许值。

1 Factor: $\dfrac{(x-5)(x+5)}{(x+5)(x-3)}$.

> 分解：$\dfrac{(x-5)(x+5)}{(x+5)(x-3)}$。

2 Cancel: $\dfrac{x-5}{x-3}$.

> 约分：$\dfrac{x-5}{x-3}$。

3 NPV: $x \neq 3$.

> 不允许值：$x \neq 3$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Using the numerator to find NPVs

用分子去找不允许值

State the NPVs of $\dfrac{x^2-4}{x^2-6x}$.

写出 $\dfrac{x^2-4}{x^2-6x}$ 的不允许值。

1 Set the top to zero: $x^2-4=(x-2)(x+2)=0$.

令分子为零：$x^2-4=(x-2)(x+2)=0$。

2 NPVs: $x\neq\pm2$.

不允许值：$x\neq\pm2$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Treating opposite factors as equal

把相反因式当成相同因式

Simplify $\dfrac{3-x}{x^2-9}$ and state the NPVs.

化简 $\dfrac{3-x}{x^2-9}$，并写出不允许值。

1 Factor: $\dfrac{3-x}{(x-3)(x+3)}$, so $x\neq\pm3$.

分解：$\dfrac{3-x}{(x-3)(x+3)}$，所以 $x\neq\pm3$。

2 $3-x$ and $x-3$ are the same, so they cancel: $\dfrac{1}{x+3}$.

$3-x$ 和 $x-3$ 一样，约掉：$\dfrac{1}{x+3}$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Sign slip when taking out a negative

提负号时符号出错

Simplify $\dfrac{8-2x}{x^2-8x+16}$ and state the NPV.

化简 $\dfrac{8-2x}{x^2-8x+16}$，并写出不允许值。

1 Bottom: $(x-4)^2$, so $x \neq 4$.

分母：$(x-4)^2$，所以 $x \neq 4$。

2 Top: $8-2x = -2(x+4)$.

分子：$8-2x = -2(x+4)$。

3 Nothing cancels: $\dfrac{-2(x+4)}{(x-4)^2}$.

没有能约的：$\dfrac{-2(x+4)}{(x-4)^2}$。

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 1 —** The x^2 is added to something, not multiplied, so it is a term, not a factor. Only factors that MULTIPLY the whole top and the whole bottom can cancel.

   x^2 是和别的式子相加的，是「项」不是「因式」。只有乘在整个分子和整个分母上的因式才能约。

   Factor first: $\dfrac{x(x+4)}{x(x+3)}$, NPVs $x \neq 0,\ -3$; cancel x: $\dfrac{x+4}{x+3}$. (At $x=1$: $\frac{5}{4}$, not $\frac{4}{3}$.)

   先分解：$\dfrac{x(x+4)}{x(x+3)}$，不允许值 $x \neq 0,\ -3$；约去 x：$\dfrac{x+4}{x+3}$。（$x=1$ 时是 $\frac{5}{4}$，不是 $\frac{4}{3}$。）

2. **line · 第 3 —** The NPV was read off the bottom AFTER $(x+5)$ was cancelled, so $x=-5$ disappeared. At $x=-5$ the original is $\frac{0}{0}$, which is undefined.

   不允许值是在约掉 $(x+5)$ 之后才看分母的，所以 $x=-5$ 丢了。$x=-5$ 时原式是 $\frac{0}{0}$，没有意义。

   Write the NPVs from the factored original bottom $(x+5)(x-3)$: $x \neq -5,\ 3$.

   从分解后的原分母 $(x+5)(x-3)$ 写不允许值：$x \neq -5,\ 3$。

3. **line · 第 1 —** A zero on top is allowed; the value of the fraction is just 0. Only division by zero is a problem, so only the bottom matters.

   分子为零是允许的，分式的值就是 0。只有除以零才出问题，所以只看分母。

   Bottom: $x^2-6x = x(x-6)$, so $x \neq 0$ and $x \neq 6$.

   分母：$x^2-6x = x(x-6)$，所以 $x \neq 0,\ x \neq 6$。

4. **line · 第 2 —** $3-x$ and $x-3$ are opposites, not equal: at $x=5$ they are -2 and 2. Opposites cancel to -1, not to 1.

   $3-x$ 和 $x-3$ 互为相反数，并不相等：$x=5$ 时分别是 -2 和 2。相反因式约成 -1，不是 1。

   $3-x = -(x-3)$, so the result is $-\dfrac{1}{x+3}$, $x \neq \pm 3$. (At $x=0$: $\frac{3}{-9} = -\frac{1}{3}$ ✓.)

   $3-x = -(x-3)$，所以结果是 $-\dfrac{1}{x+3}$，$x \neq \pm 3$。（$x=0$ 时：$\frac{3}{-9} = -\frac{1}{3}$ ✓。）

5. **line · 第 2 —** Taking out -2 flips the sign of EVERY term inside: $-2(x+4) = -2x-8$, not $8-2x$.

   提出 -2 时，括号里「每一项」都要变号：$-2(x+4) = -2x-8$，不是 $8-2x$。

   $8-2x = -2(x-4)$ (check: $-2x+8$ ✓). Cancel one $(x-4)$: $-\dfrac{2}{x-4}$, $x \neq 4$.

   $8-2x = -2(x-4)$（检验：$-2x+8$ ✓）。约去一个 $(x-4)$：$-\dfrac{2}{x-4}$，$x \neq 4$。