Multiplying & dividing

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Is it $\times$ or $\div$? Factor every numerator and every denominator before anything else.
2. 🎯 **Asked** One fraction in lowest terms, followed by EVERY non-permissible value.
3. ✏️ **Picture it** Draw three boxes for NPVs: bottom of fraction 1, bottom of fraction 2, and (for division only) top of the divisor. Fill all three before you flip.
4. 🧰 **Tool** Division: keep the first fraction, change $\div$ to $\times$, flip the SECOND fraction. Then cancel any top factor with any bottom factor and multiply what is left.
5. ✅ **Check** Test an allowed value such as $x = 0$ or $x = 1$ in the original and in your answer. Count your NPVs against the three boxes.

⭐ Must know

- Multiply: factor everything, cancel any top with any bottom, then multiply what is left.
- Divide: multiply by the reciprocal of the SECOND fraction.
- Division NPVs: both denominators AND the numerator of the divisor.

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$$

🔍 Watch out for

1. Missing the NPV from the divisor's numerator
2. Cancelling across a plus sign
3. Losing the divisor's old denominator after flipping
4. Cancelling a power completely
5. Flipping the wrong fraction

🌱 Small wins — tick as you go

- ○ I can divide number fractions, factor a trinomial and simplify $\frac{x^5}{x^2}$. (5′)
- ○ I can ask the five starting questions for a product or quotient of rational expressions. (5′)
- ○ I can follow, then finish, a "factor, list NPVs from three places, flip, cancel" division. (8′)
- ○ I can do a division on my own with at most two hints and get every NPV. (9′)
- ○ I can spot the five classic mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Compute $\frac{2}{3} \div \frac{4}{9}$.

A. $\frac{8}{27}$ B. $\frac{3}{2}$ C. $\frac{2}{3}$ D. $\frac{6}{12}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.

2. Factor $x^2 + 2x - 15$.

A. $(x-5)(x+3)$ B. $(x+5)(x+3)$ C. $(x+5)(x-3)$ D. $(x+15)(x-1)$

If you missed it — **Factoring $x^2 + bx + c$ (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12$ → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x+2)(x+5)$.
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.

3. Simplify $\frac{x^5}{x^2}$.

A. x^3 B. $x^{2.5}$ C. x^7 D. x^{10}

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3 x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

4. Compute $\frac{3}{4} \cdot \frac{8}{9}$.

A. $\frac{11}{13}$ B. $\frac{27}{32}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
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3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.

Answers: 1: B · 2: C · 3: A · 4: C

🔍 Common-errors sheet

1. Missing the NPV from the divisor's numerator

Simplify $\dfrac{x-5}{x+2} \div \dfrac{x-5}{x-1}$ and state all NPVs.

1 NPVs: the bottoms give $x \neq -2$ and $x \neq 1$. That is all of them.

2 Flip and cancel: $\dfrac{x-5}{x+2} \cdot \dfrac{x-1}{x-5} = \dfrac{x-1}{x+2}$.

✍ Which line goes wrong first, and why?

2. Cancelling across a plus sign

Simplify $\dfrac{x+5}{x} \cdot \dfrac{2x}{x^2-25}$.

1 Cancel the x in $\dfrac{x+5}{x}$: it becomes 5.

2 $5 \cdot \dfrac{2x}{x^2-25} = \dfrac{10x}{x^2-25}$.

✍ Which line goes wrong first, and why?

3. Losing the divisor's old denominator after flipping

Simplify $\dfrac{2x}{x-3} \div \dfrac{x+1}{x+4}$ and state all NPVs.

1 Flip: $\dfrac{2x}{x-3} \cdot \dfrac{x+4}{x+1} = \dfrac{2x(x+4)}{(x-3)(x+1)}$.

2 NPVs from this bottom: $x \neq 3,\ -1$.

✍ Which line goes wrong first, and why?

4. Cancelling a power completely

Simplify $\dfrac{x^2-9}{4x} \cdot \dfrac{8x^3}{x+3}$.

1 Factor: $\dfrac{(x-3)(x+3)}{4x} \cdot \dfrac{8x^3}{x+3}$.

2 Cancel $(x+3)$, 4 into 8, and x into x^3: $2x(x-3)$.

✍ Which line goes wrong first, and why?

5. Flipping the wrong fraction

Simplify $\dfrac{x}{x+3} \div \dfrac{x^2}{x^2-9}$ and state all NPVs.

1 Flip the first one: $\dfrac{x+3}{x} \cdot \dfrac{x^2}{x^2-9}$.

2 Cancel: $\dfrac{x}{x-3}$.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1 —** After flipping, the divisor's top $x-5$ lands on the bottom. At $x=5$ you would be dividing by $\frac{0}{4}=0$, which is not allowed. Three boxes: $x \neq -2$, $x \neq 1$, and $x \neq 5$ from the divisor top. Answer $\dfrac{x-1}{x+2}$, $x \neq -2,\ 1,\ 5$.
2. **line 1 —** In $x+5$ the x is a term that is ADDED, not a factor of the whole top. At $x=1$, $\frac{x+5}{x}=6$, not 5. Factor: $\dfrac{x+5}{x} \cdot \dfrac{2x}{(x-5)(x+5)}$; cancel x (top of one, bottom of the other) and $(x+5)$: $\dfrac{2}{x-5}$, $x \neq 0,\ \pm 5$.
3. **line 2 —** Reading the NPVs from the final bottom misses $x+4$: it was a denominator in the ORIGINAL question and moved to the top when it was flipped. At $x=-4$ the original has $\frac{-3}{0}$. List NPVs before flipping: $x \neq 3$, $x \neq -4$, and $x \neq -1$ from the divisor top.
4. **line 2 —** $\dfrac{x^3}{x}=x^2$: one x cancels and two are left. Writing x subtracts 2 from the exponent instead of 1. $\dfrac{8x^3}{4x}=2x^2$, so the answer is $2x^2(x-3)$, $x \neq 0,\ -3$.
5. **line 1 —** Dividing by a fraction means multiplying by the reciprocal of the DIVISOR, the second fraction. Flipping the first one computes its upside-down value instead. $\dfrac{x}{x+3} \cdot \dfrac{(x-3)(x+3)}{x^2}=\dfrac{x-3}{x}$. NPVs: $x \neq -3$, $x \neq \pm 3$, and $x \neq 0$ from the divisor top x^2.