Adding & subtracting

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Factor every denominator first and write the NPVs from them (every value that makes any denominator 0).
2. 🎯 **Asked** ONE fraction, numerator and denominator factored, with nothing left to cancel, plus the NPVs.
3. ✏️ **Picture it** Line the factored denominators up in a row. Under each one, write the factors it is missing from the LCD.
4. 🧰 **Tool** Multiply each numerator by its missing factors. Write the whole new numerator on one line with brackets, then expand and collect.
5. ✅ **Check** Factor the numerator and cancel common FACTORS only. Then put one allowed value (like $x = 0$) into the question and your answer — they must match.

⭐ Must know

- Factor every denominator; LCD = each different factor at its highest power.
- Rewrite over the LCD, then add or subtract the NUMERATORS only.
- Subtracting: bracket the whole numerator, $-(x - 2) = -x + 2$; finish by factoring and cancelling.

$$\frac{A}{B} \pm \frac{C}{D} = \frac{AD \pm BC}{BD}$$

🔍 Watch out for

1. Adding the denominators too
2. The minus sign only hits the first term
3. Changing the bottom but not the top
4. Treating opposite denominators as the same
5. Cancelling terms instead of factors

🌱 Small wins — tick as you go

- ○ I can subtract number fractions, find an LCD like $12x^2$, and distribute a minus sign over a bracket. (5′)
- ○ I can factor every denominator, write the NPVs, and build the LCD before touching the numerators. (5′)
- ○ I can follow, then finish, a subtraction over a binomial LCD with the second numerator in brackets. (9′)
- ○ I can subtract two rational expressions on my own (with at most two hints) and cancel at the end. (8′)
- ○ I can spot the five classic adding/subtracting mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions and test each answer with one value of x. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Work out $\frac{3}{4} - \frac{1}{6}$.
A. $\frac{2}{12}$ B. $\frac{7}{12}$ C. $\frac{1}{2}$ D. $\frac{5}{12}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.

2. What is the lowest common denominator of $\frac{1}{4x}$ and $\frac{1}{6x^2}$?
A. $24x^3$ B. $12x^3$ C. $12x^2$ D. $2x$

If you missed it — **Lowest common denominator**: Factor each denominator; the LCD uses every factor the HIGHEST number of times it appears. For 6 and 4: $2 \cdot 3$ and 2^2 → $2^2 \cdot 3 = 12$. For x and x^2: x^2. For $(x+1)$ and $(x-2)$: $(x+1)(x-2)$.

1. $\frac{1}{2x}$ and $\frac{1}{3x^2}$: LCD $= 6x^2$.
2. Multiply top AND bottom by what is missing: $\frac{1}{2x} = \frac{3x}{6x^2}$.

3. Simplify $x - (x - 5)$.
A. 5 B. -5 C. $2x - 5$ D. $-2x + 5$

If you missed it — **Distributing (and the minus sign)**: Multiply EVERY term inside the bracket: $3(x-4) = 3x - 12$. A minus in front changes every sign inside: $-(x-5) = -x + 5$, and $2 - (x+3) = 2 - x - 3 = -x - 1$.

1. $-2(3x - 1) = -6x + 2$ (the -1 becomes $+2$).
2. $x(x+2) - 3(x+2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$.

4. Factor $x^2 - x - 6$.
A. $(x-6)(x+1)$ B. $(x+3)(x-2)$ C. $(x-1)(x+6)$ D. $(x-3)(x+2)$

If you missed it — **Factoring $x^2 + bx + c$ (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12$ → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x+2)(x+5)$.
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.

Answers: 1: B · 2: C · 3: A · 4: D

🔍 Common-errors sheet

1. Adding the denominators too

Simplify $\frac{1}{x} + \frac{1}{3}$.

1 Add the tops and add the bottoms: $\frac{1+1}{x+3}$

2 $= \frac{2}{x+3}$

✍ Which line goes wrong first, and why?

2. The minus sign only hits the first term

Simplify $\frac{2x+1}{x-3} - \frac{x-4}{x-3}$.

1 Same denominator: $\frac{2x+1-x-4}{x-3}$

2 $= \frac{x-3}{x-3} = 1$

✍ Which line goes wrong first, and why?

3. Changing the bottom but not the top

Simplify $\frac{3}{x} + \frac{2}{x-1}$.

1 LCD $x(x-1)$: $\frac{3}{x(x-1)} + \frac{2}{x(x-1)}$

2 $= \frac{5}{x(x-1)}$

✍ Which line goes wrong first, and why?

4. Treating opposite denominators as the same

Simplify $\dfrac{5}{x-2}+\dfrac{3}{2-x}$.

1 The denominators look the same, so add the tops: $\dfrac{5+3}{x-2}$

2 $= \dfrac{8}{x-2}$

✍ Which line goes wrong first, and why?

5. Cancelling terms instead of factors

Simplify $\dfrac{x}{x+3}+\dfrac{6}{x+3}$.

1 Same denominator: $\dfrac{x+6}{x+3}$

2 Cancel the x on top and bottom: $\dfrac{6}{3}=2$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1** — It treats fractions like pairs of numbers. Even $\frac{1}{2}+\frac{1}{2}$ would become $\frac{2}{4}$ that way, which is clearly wrong. LCD $3x$: $\dfrac{3}{3x}+\dfrac{x}{3x}=\dfrac{x+3}{3x}$, $x\neq 0$.
2. **line 1** — Without brackets the minus was attached to x only. It must change the sign of the whole second numerator: $-(x-4)=-x+4$. $\dfrac{(2x+1)-(x-4)}{x-3}=\dfrac{x+5}{x-3}$, $x\neq 3$.
3. **line 1** — The denominators were made bigger but the numerators were not multiplied by the same factor, so each fraction changed its value. $\dfrac{3(x-1)+2x}{x(x-1)}=\dfrac{5x-3}{x(x-1)}$, $x\neq 0,\ 1$.
4. **line 1** — $2-x$ and $x-2$ have the same numbers but opposite signs: $2-x=-(x-2)$. At $x=3$ one is 1, the other -1. $\dfrac{3}{2-x}=-\dfrac{3}{x-2}$, so $\dfrac{5-3}{x-2}=\dfrac{2}{x-2}$, $x\neq 2$.
5. **line 2** — The x here is a term joined by $+$, not a factor joined by $\times$. You may only cancel something that multiplies the WHOLE top and the WHOLE bottom. $x+6$ and $x+3$ share no factor, so $\dfrac{x+6}{x+3}$ is already simplest. (Test $x=1$: $\frac{7}{4}\neq 2$.)