Adding & subtracting

有理式加减

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Factor every denominator first and write the NPVs from them (every value that makes any denominator 0).
   > 先把每个分母分解，并据此写出不允许值（凡是让任何一个分母为 0 的值）。
2. 🎯 **Asked · 求什么** ONE fraction, numerator and denominator factored, with nothing left to cancel, plus the NPVs.
   > 一个分式：分子、分母都分解好，不能再约分，再加上不允许值。
3. ✏️ **Picture it · 画出来** Line the factored denominators up in a row. Under each one, write the factors it is missing from the LCD.
   > 把分解后的分母排成一行，在每个分母下面写出它比公分母少了哪些因式。
4. 🧰 **Tool · 用什么工具** Multiply each numerator by its missing factors. Write the whole new numerator on one line with brackets, then expand and collect.
   > 每个分子乘上自己缺的因式。先在一行写出带括号的整个新分子，再展开、合并同类项。
5. ✅ **Check · 检验** Factor the numerator and cancel common FACTORS only. Then put one allowed value (like $x = 0$) into the question and your answer — they must match.
   > 把分子分解，只约公「因式」。再取一个允许值（比如 $x = 0$）代入原题和答案——两边必须相等。

⭐ Must know

必须记住

- Factor every denominator; LCD = each different factor at its highest power.
  > 把每个分母分解；最简公分母＝每个不同因式取最高次幂。
- Rewrite over the LCD, then add or subtract the NUMERATORS only.
  > 化成同分母，只对分子加减。
- Subtracting: bracket the whole numerator, $-(x-2) = -x+2$; finish by factoring and cancelling.
  > 相减时整个分子加括号，$-(x-2) = -x+2$；最后分解分子看能否约分。

$$\frac{A}{B} \pm \frac{C}{D} = \frac{AD \pm BC}{BD}$$

🔍 Watch out for

当心

1. Adding the denominators too
   > 连分母也相加
2. The minus sign only hits the first term
   > 减号只作用到第一项
3. Changing the bottom but not the top
   > 只改了分母，没改分子
4. Treating opposite denominators as the same
   > 把相反的分母当成相同的
5. Cancelling terms instead of factors
   > 约掉了「项」而不是「因式」

🌱 Small wins — tick as you go

一小步一小步——做完打勾

- ○ I can subtract number fractions, find an LCD like $12x^2$, and distribute a minus sign over a bracket.
  > 我会算数字分数的减法，会找 $12x^2$ 这样的最简公分母，会把减号分配进括号。

  (5′)
- ○ I can factor every denominator, write the NPVs, and build the LCD before touching the numerators.
  > 我能先把每个分母分解、写出不允许值、建好最简公分母，再去管分子。

  (5′)
- ○ I can follow, then finish, a subtraction over a binomial LCD with the second numerator in brackets.
  > 我能看懂并补全以二项式乘积为公分母的减法，第二个分子记得加括号。

  (9′)
- ○ I can subtract two rational expressions on my own (with at most two hints) and cancel at the end.
  > 我能在最多两个提示下独立完成两个分式相减，并在最后约分。

  (8′)
- ○ I can spot the five classic adding/subtracting mistakes in someone else's work.
  > 我能在别人的解答里找出分式加减的五个经典错误。

  (8′)
- ○ I can do the Level 1–2 practice questions and test each answer with one value of x.
  > 我能做出 1–2 级练习题，并用一个 x 值检验每个答案。

  (10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Work out $\frac{3}{4} - \frac{1}{6}$.

> 计算 $\frac{3}{4} - \frac{1}{6}$。

A. $\frac{2}{12}$ B. $\frac{7}{12}$ C. $\frac{1}{2}$ D. $\frac{5}{12}$

If you missed it · 如果做错了 — **Fractions: add, subtract, multiply, divide**

> **分数的四则运算**

: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

> 加减必须先通分：$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$。乘法分子乘分子、分母乘分母。除以一个分数＝乘它的倒数（把第二个分数倒过来）。

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
   > $\frac{2}{3} - \frac{1}{4}$：公分母 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$。
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
   > $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$。
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.
   > 分母不能相加：$\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$。

2. What is the lowest common denominator of $\frac{1}{4x}$ and $\frac{1}{6x^2}$?

> $\frac{1}{4x}$ 和 $\frac{1}{6x^2}$ 的最简公分母是什么?

A. $24x^3$ B. $12x^3$ C. $12x^2$ D. $2x$

If you missed it · 如果做错了 — **Lowest common denominator**

> **最简公分母**

: Factor each denominator; the LCD uses every factor the HIGHEST number of times it appears. For 6 and 4: $2 \cdot 3$ and 2^2 → $2^2 \cdot 3 = 12$. For x and x^2: x^2. For $(x+1)$ and $(x-2)$: $(x+1)(x-2)$.

> 把每个分母分解；最简公分母取每个因式出现的最高次数。6 和 4：$2 \cdot 3$ 与 2^2 → $2^2 \cdot 3 = 12$。x 和 x^2：x^2。$(x+1)$ 和 $(x-2)$：$(x+1)(x-2)$。

1. $\frac{1}{2x}$ and $\frac{1}{3x^2}$: LCD $= 6x^2$.
   > $\frac{1}{2x}$ 与 $\frac{1}{3x^2}$：最简公分母 $= 6x^2$。
2. Multiply top AND bottom by what is missing: $\frac{1}{2x} = \frac{3x}{6x^2}$.
   > 分子分母同乘缺的部分：$\frac{1}{2x} = \frac{3x}{6x^2}$。

3. Simplify $x-(x-5)$.

> 化简 $x-(x-5)$。

A. 5 B. -5 C. $2x-5$ D. $-2x+5$

If you missed it · 如果做错了 — **Distributing (and the minus sign)**

> **去括号（尤其是负号）**

: Multiply EVERY term inside the bracket: $3(x-4)=3x-12$. A minus in front changes every sign inside: $-(x-5)=-x+5$, and $2-(x+3)=2-x-3=-x-1$.

> 括号里的每一项都要乘到：$3(x-4)=3x-12$。前面是负号时，括号里每一项都变号：$-(x-5)=-x+5$，$2-(x+3)=2-x-3=-x-1$。

1. $-2(3x-1)=-6x+2$ (the -1 becomes $+2$).
   > $-2(3x-1)=-6x+2$（-1 变成 $+2$）。
2. $x(x+2)-3(x+2)=x^2+2x-3x-6=x^2-x-6$.
   > $x(x+2)-3(x+2)=x^2+2x-3x-6=x^2-x-6$。

4. Factor x^2-x-6.

> 分解因式 x^2-x-6。

A. $(x-6)(x+1)$ B. $(x+3)(x-2)$ C. $(x-1)(x+6)$ D. $(x-3)(x+2)$

If you missed it · 如果做错了 — **Factoring x^2+bx+c (product and sum)**

> **分解 x^2+bx+c（积与和）**

: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: x^2-x-12 → -4 and 3 → $(x-4)(x+3)$. Check by expanding.

> 先提公因式。再找两个数，相乘等于 c、相加等于 b：x^2-x-12 → -4 和 3 → $(x-4)(x+3)$。展开检验。

1. $x^2+7x+10$: $2\times5=10$, $2+5=7$ → $(x+2)(x+5)$.
   > $x^2+7x+10$：$2\times5=10$，$2+5=7$ → $(x+2)(x+5)$。
2. If $c<0$ the two numbers have different signs; if $c>0$ they share the sign of b.
   > 若 $c<0$，两个数异号；若 $c>0$，两个数与 b 同号。

Answers · 答案: 1: B · 2: C · 3: A · 4: D

🔍 Common-errors sheet

常见错误单

1. Adding the denominators too

连分母也相加

Simplify $\frac{1}{x}+\frac{1}{3}$.

化简 $\frac{1}{x}+\frac{1}{3}$。

1 Add the tops and add the bottoms: $\frac{1+1}{x+3}$

分子加分子、分母加分母：$\frac{1+1}{x+3}$

2 $=\frac{2}{x+3}$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. The minus sign only hits the first term

减号只作用到第一项

Simplify $\frac{2x+1}{x-3}-\frac{x-4}{x-3}$.

化简 $\frac{2x+1}{x-3}-\frac{x-4}{x-3}$。

1 Same denominator: $\frac{2x+1-x-4}{x-3}$

同分母：$\frac{2x+1-x-4}{x-3}$

2 $=\frac{x-3}{x-3}=1$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Changing the bottom but not the top

> **只改了分母，没改分子**

Simplify $\frac{3}{x}+\frac{2}{x-1}$.

> 化简 $\frac{3}{x}+\frac{2}{x-1}$。

1 LCD $x(x-1)$: $\frac{3}{x(x-1)}+\frac{2}{x(x-1)}$

> 公分母 $x(x-1)$：$\frac{3}{x(x-1)}+\frac{2}{x(x-1)}$

2 $=\frac{5}{x(x-1)}$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Treating opposite denominators as the same

> **把相反的分母当成相同的**

Simplify $\frac{5}{x-2}+\frac{3}{2-x}$.

> 化简 $\frac{5}{x-2}+\frac{3}{2-x}$。

1 The denominators look the same, so add the tops: $\frac{5+3}{x-2}$

> 分母看起来一样，直接加分子：$\frac{5+3}{x-2}$

2 $=\frac{8}{x-2}$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Cancelling terms instead of factors

> **约掉了「项」而不是「因式」**

Simplify $\dfrac{x}{x+3}+\dfrac{6}{x+3}$.

> 化简 $\dfrac{x}{x+3}+\dfrac{6}{x+3}$。

1 Same denominator: $\dfrac{x+6}{x+3}$

> 同分母：$\dfrac{x+6}{x+3}$

2 Cancel the x on top and bottom: $\dfrac{6}{3}=2$

> 分子分母的 x 约掉：$\dfrac{6}{3}=2$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 1 —** It treats fractions like pairs of numbers. Even $\frac{1}{2}+\frac{1}{2}$ would become $\frac{2}{4}$ that way, which is clearly wrong.
   > 把分式当成一对对的数来加。照这样 $\frac{1}{2}+\frac{1}{2}$ 会变成 $\frac{2}{4}$，显然不对。

   LCD $3x$: $\dfrac{3}{3x}+\dfrac{x}{3x}=\dfrac{x+3}{3x}$, $x\neq 0$.
   > 公分母 $3x$：$\dfrac{3}{3x}+\dfrac{x}{3x}=\dfrac{x+3}{3x}$，$x\neq 0$。
2. **line · 第 1 —** Without brackets the minus was attached to x only. It must change the sign of the whole second numerator: $-(x-4)=-x+4$.
   > 没加括号，减号只给了 x。它要改变整个第二个分子的符号：$-(x-4)=-x+4$。

   $\dfrac{(2x+1)-(x-4)}{x-3}=\dfrac{x+5}{x-3}$, $x\neq 3$.
   > $\dfrac{(2x+1)-(x-4)}{x-3}=\dfrac{x+5}{x-3}$，$x\neq 3$。
3. **line · 第 1 —** The denominators were made bigger but the numerators were not multiplied by the same factor, so each fraction changed its value.
   > 分母变大了，分子却没乘同一个因式，于是每个分式的值都变了。

   $\dfrac{3(x-1)+2x}{x(x-1)}=\dfrac{5x-3}{x(x-1)}$, $x\neq 0,\ 1$.
   > $\dfrac{3(x-1)+2x}{x(x-1)}=\dfrac{5x-3}{x(x-1)}$，$x\neq 0,\ 1$。
4. **line · 第 1 —** $2-x$ and $x-2$ have the same numbers but opposite signs: $2-x=-(x-2)$. At $x=3$ one is 1, the other -1.
   > $2-x$ 和 $x-2$ 数字一样，符号相反：$2-x=-(x-2)$。$x=3$ 时一个是 1，一个是 -1。

   $\dfrac{3}{2-x}=-\dfrac{3}{x-2}$, so $\dfrac{5-3}{x-2}=\dfrac{2}{x-2}$, $x\neq 2$.
   > $\dfrac{3}{2-x}=-\dfrac{3}{x-2}$，所以 $\dfrac{5-3}{x-2}=\dfrac{2}{x-2}$，$x\neq 2$。
5. **line · 第 2 —** The x here is a term joined by $+$, not a factor joined by $\times$. You may only cancel something that multiplies the WHOLE top and the WHOLE bottom.
   > 这里的 x 是用「+」连着的项，不是用「×」连着的因式。只有乘在整个分子和整个分母上的东西才能约。

   $x+6$ and $x+3$ share no factor, so $\dfrac{x+6}{x+3}$ is already simplest. (Test $x=1$: $\frac{7}{4}\neq 2$.)
   > $x+6$ 与 $x+3$ 没有公因式，$\dfrac{x+6}{x+3}$ 已经最简。(代 $x=1$：$\frac{7}{4}\neq 2$。)