Factoring trinomials ax²+bx+c

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Write a, b, c with their signs. First look for a GCF (and a negative leading coefficient) and take it out.
2. 🎯 **Asked** Two brackets that multiply back to the original. "Factor fully" means nothing inside can be factored again.
3. ✏️ **Picture it** A small sign table: product ac positive → same signs (the sign of b); negative → opposite signs, the bigger one takes the sign of b.
4. 🧰 **Tool** If $a = 1$, find two numbers with product c and sum b. If $a \neq 1$, product ac and sum b, split the middle term, then group.
5. ✅ **Check** Expand your brackets — at least the middle term (outer + inner). It must match bx exactly, sign included.

⭐ Must know

- $a = 1$: two numbers that multiply to c and add to b.
- $a \neq 1$: two numbers that multiply to ac and add to b; split the middle, then group.
- Signs: $c > 0$ → both take the sign of b; $c < 0$ → opposite signs, bigger one takes the sign of b.

$$x^2 + (m+n)x + mn = (x+m)(x+n)$$

$ax^2 + bx + c$: $\quad mn = ac, \quad m + n = b \;\Rightarrow\; ax^2 + mx + nx + c$

🔍 Watch out for

1. Signs swapped in the brackets
2. Searching on c instead of a times c
3. Stopping before the GCF is out
4. Checking the product but not the sum
5. Losing the minus sign when grouping

🌱 Small wins — tick as you go

- ○ I can add and multiply negative numbers, expand two binomials, and factor $x^2 + bx + c$ when the numbers are small. (6′)
- ○ I can write a, b, c and ac in the margin and decide the signs of the two numbers before searching. (6′)
- ○ I can follow, then finish, a decomposition: find the pair, split the middle term, group, check. (10′)
- ○ I can factor an $ax^2 + bx + c$ trinomial on my own with at most two hints. (8′)
- ○ I can spot the five classic factoring mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions and check each one by expanding. (12′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Work out $(-9) + 4$.
A. -13 B. -5 C. 5 D. 13

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

2. Expand $(2x + 3)(x - 4)$.
A. $2x^2 - 12$ B. $2x^2 + 5x - 12$ C. $2x^2 - 5x - 12$ D. $2x^2 - 11x - 12$

If you missed it — **Multiplying binomials,** $(a + b)^2$: Every term times every term (FOIL): $(x + 3)(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$. A square has a middle term: $(x + 3)^2 = x^2 + 6x + 9$, never $x^2 + 9$.

1. $(x - 4)^2 = (x - 4)(x - 4) = x^2 - 4x - 4x + 16 = x^2 - 8x + 16$.
2. Shortcut: $(a \pm b)^2 = a^2 \pm 2ab + b^2$ — middle term $= 2\times$ first $\times$ last.

3. Factor $x^2 + 7x + 10$.
A. $(x + 2)(x + 5)$ B. $(x + 1)(x + 10)$ C. $(x + 3)(x + 4)$ D. $(x - 2)(x - 5)$

If you missed it — **Factoring** $x^2 + bx + c$ **(product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12$ → -4 and 3 → $(x - 4)(x + 3)$. Check by expanding.

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x + 2)(x + 5)$.
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.

4. Expand $-3(x - 5)$.
A. $-3x - 15$ B. $3x + 15$ C. $-3x - 5$ D. $-3x + 15$

If you missed it — **Distributing (and the minus sign)**: Multiply EVERY term inside the bracket: $3(x - 4) = 3x - 12$. A minus in front changes every sign inside: $-(x - 5) = -x + 5$, and $2 - (x + 3) = 2 - x - 3 = -x - 1$.

1. $-2(3x - 1) = -6x + 2$ (the -1 becomes $+2$).
2. $x(x + 2) - 3(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$.

Answers: 1: B · 2: C · 3: A · 4: D

🔍 Common-errors sheet

1. Signs swapped in the brackets

Factor $x^2 - 2x - 24$.

1 Product -24, sum -2: the numbers are 6 and 4.

2 $(x + 6)(x - 4)$

✍ Which line goes wrong first, and why?

2. Searching on c instead of a times c

Factor $3x^2 + 11x + 6$.

1 Need product 6 and sum 11.

2 Pairs of 6 add to 7 or 5 — none adds to 11, so it does not factor.

✍ Which line goes wrong first, and why?

3. Stopping before the GCF is out

Factor fully: $3x^2 + 15x + 18$.

1 $ac = 54$, sum 15: 9 and 6.

2 $3x^2 + 9x + 6x + 18 = 3x(x + 3) + 6(x + 3)$

3 $= (x + 3)(3x + 6)$. Done.

✍ Which line goes wrong first, and why?

4. Checking the product but not the sum

Factor $x^2 + 5x - 24$.

1 $12 \cdot (-2) = -24$ ✓

2 So $(x + 12)(x - 2)$.

✍ Which line goes wrong first, and why?

5. Losing the minus sign when grouping

Factor $6x^2 + x - 2$.

1 $ac = -12$, sum 1: 4 and -3. Split: $6x^2 + 4x - 3x - 2$.

2 Group: $2x(3x + 2) - 1(3x - 2)$

3 The brackets differ, so it does not factor.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** The right pair was found but the minus went on the wrong one. $(x + 6)(x - 4)$ has middle term $+2x$. The bigger number takes the sign of $b = -2$: -6 and $+4$, so $(x - 6)(x + 4)$. Check: $4x - 6x = -2x$ ✓.
2. **line 1 —** The $a = 1$ shortcut was used when $a = 3$. With $a \neq 1$ the product must be ac. $ac = 18$, sum 11: 9 and 2. $3x^2 + 9x + 2x + 6 = 3x(x + 3) + 2(x + 3) = (x + 3)(3x + 2)$.
3. **line 3 —** The brackets multiply back correctly, so it felt finished. But $3x + 6$ still has a common factor 3; skipping the GCF at the start is what let it hide. GCF first: $3(x^2 + 5x + 6) = 3(x + 2)(x + 3)$.
4. **line 2 —** The first pair with the right product was taken. But $12 + (-2) = 10$, not 5 — both conditions must hold. List pairs of -24 and their sums: 8 and -3 give 5. So $(x + 8)(x - 3)$.
5. **line 2 —** Taking out -1 changes EVERY sign in the group: $-3x - 2 = -1(3x + 2)$. Only the first sign was changed. $2x(3x + 2) - 1(3x + 2) = (3x + 2)(2x - 1)$. Check: $-3x + 4x = x$ ✓.