Difference of squares & perfect squares

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Count the terms. Is there a common factor? Write each square term as $(\text{something})^2$, and note the sign between them.
2. 🎯 **Asked** A product of brackets that cannot be split any further over the integers ("factor fully").
3. ✏️ **Picture it** A big square of side A with a small square of side B cut from the corner: the leftover L-shape rearranges into an $(A-B)$ by $(A+B)$ rectangle. A trinomial $(A+B)^2$ is the big square split into A^2, B^2 and two AB strips.
4. 🧰 **Tool** Common factor first. Two squares with a minus: $(A-B)(A+B)$. Three terms with square ends: test whether the middle is exactly $2AB$; if yes $(A \pm B)^2$, if not factor it as an ordinary trinomial.
5. ✅ **Check** Expand your answer and get the original back. Then look at every bracket once more: is any of them still a difference of squares?

⭐ Must know

- $A^2 - B^2 = (A-B)(A+B)$: two squares with a MINUS.
- $A^2 \pm 2AB + B^2 = (A \pm B)^2$: check the middle is exactly $2AB$.
- $A^2 + B^2$ does not factor over the reals; keep going on $x^4 - 16$.

$$A^2 - B^2 = (A-B)(A+B)$$

$$A^2 \pm 2AB + B^2 = (A \pm B)^2$$

🔍 Watch out for

1. Factoring a sum of squares
2. Stopping before it is fully factored
3. Writing a difference of squares as a perfect square
4. Calling it a perfect square without checking the middle
5. Not looking for a common factor first

🌱 Small wins — tick as you go

- ○ I can expand $(A+B)^2$ and $(A-B)(A+B)$, and I can see that 64 and $9x^4$ are perfect squares. (5′)
- ○ I can ask the five starting questions for a two-term or three-term expression. (5′)
- ○ I can follow, then finish, a "common factor first, then difference of squares twice" factorization. (8′)
- ○ I can factor a fourth-power expression fully on my own with at most two hints. (8′)
- ○ I can spot the five classic mistakes in someone else's factoring. (8′)
- ○ I can do the Level 1–2 practice questions. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Expand $(2x+5)^2$.
A. $4x^2+25$ B. $4x^2+10x+25$ C. $4x^2+20x+25$ D. $2x^2+20x+25$

If you missed it — **Multiplying binomials,** $(a+b)^2$: Every term times every term (FOIL): $(x+3)(x-5)=x^2-5x+3x-15=x^2-2x-15$. A square has a middle term: $(x+3)^2=x^2+6x+9$, never x^2+9.

1. $(x-4)^2=(x-4)(x-4)=x^2-4x-4x+16=x^2-8x+16$.
2. Shortcut: $(a\pm b)^2=a^2\pm 2ab+b^2$ — middle term $=2\times$ first $\times$ last.

2. Expand $(x-6)(x+6)$.
A. x^2-36 B. x^2+36 C. $x^2-12x-36$ D. $x^2-6x+36$

If you missed it — **Distributing (and the minus sign)**: Multiply EVERY term inside the bracket: $3(x-4)=3x-12$. A minus in front changes every sign inside: $-(x-5)=-x+5$, and $2-(x+3)=2-x-3=-x-1$.

1. $-2(3x-1)=-6x+2$ (the -1 becomes $+2$).
2. $x(x+2)-3(x+2)=x^2+2x-3x-6=x^2-x-6$.

3. Which number is a perfect square?
A. 18 B. 32 C. 50 D. 64

If you missed it — **Perfect squares and square roots**: Know the squares $1,4,9,\ldots,225$. $\sqrt{ab}=\sqrt{a}\sqrt{b}$, so $\sqrt{50}=\sqrt{25}\sqrt{2}=5\sqrt{2}$. But $\sqrt{a+b}\neq\sqrt{a}+\sqrt{b}$: $\sqrt{9+16}=5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2=49$ has TWO answers $x=\pm 7$, but $\sqrt{49}=7$ only.

4. Simplify $(3x^2)^2$.
A. $3x^4$ B. $9x^4$ C. $6x^4$ D. $9x^2$

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3x^4=x^7$); divide → SUBTRACT ($x^7\div x^2=x^5$); power of a power → MULTIPLY ($(x^3)^2=x^6$). A power of a product hits every factor: $(2x)^3=8x^3$.

1. $(3x^2)^2\cdot x=9x^4\cdot x=9x^5$.
2. $\frac{6x^5}{2x^2}=3x^3$ (divide the numbers, subtract the exponents).

Answers: 1: C · 2: A · 3: D · 4: B

🔍 Common-errors sheet

1. Factoring a sum of squares

Factor fully: $16x^2 + 25$.

1 Both terms are squares: $(4x)^2$ and 5^2.

2 So $16x^2 + 25 = (4x - 5)(4x + 5)$.

✍ Which line goes wrong first, and why?

2. Stopping before it is fully factored

Factor fully: $16x^4 - 1$.

1 $16x^4 = (4x^2)^2$ and $1 = 1^2$.

2 $16x^4 - 1 = (4x^2 - 1)(4x^2 + 1)$. Done.

✍ Which line goes wrong first, and why?

3. Writing a difference of squares as a perfect square

Factor $9x^2 - 49$.

1 $9x^2 = (3x)^2$ and $49 = 7^2$, with a minus between.

2 So $9x^2 - 49 = (3x - 7)^2$.

✍ Which line goes wrong first, and why?

4. Calling it a perfect square without checking the middle

Factor $x^2 + 13x + 36$.

1 $x^2 = (x)^2$ and $36 = 6^2$, so this is a perfect square.

2 $x^2 + 13x + 36 = (x + 6)^2$.

✍ Which line goes wrong first, and why?

5. Not looking for a common factor first

Factor fully: $12x^2 - 75$.

1 12 is not a perfect square, and neither is 75.

2 So $12x^2 - 75$ cannot be factored.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** The pattern $(A - B)(A + B)$ needs a MINUS between the squares. Expanding $(4x - 5)(4x + 5)$ gives $16x^2 - 25$, not $+25$. A sum of two squares has no real factors, and 16 and 25 share no common factor, so it stays as it is.
2. **line 2 —** The first round is right, but "factor fully" means checking each bracket again. $4x^2 - 1 = (2x)^2 - 1^2$ is another difference of squares. $4x^2 - 1 = (2x - 1)(2x + 1)$; $4x^2 + 1$ is a sum and stays. So $(2x - 1)(2x + 1)(4x^2 + 1)$.
3. **line 2 —** The minus sign got copied into both brackets. $(3x - 7)^2 = 9x^2 - 42x + 49$ has a middle term and ends in $+49$; a difference of squares needs one minus bracket and one plus bracket. $9x^2 - 49 = (3x - 7)(3x + 7)$. Expand to check: $9x^2 + 21x - 21x - 49$ ✓.
4. **line 1 —** Square ends are not enough. For $(x + 6)^2$ the middle would be $2 \cdot x \cdot 6 = 12x$, but here it is $13x$. Treat it as an ordinary trinomial: product 36, sum 13 gives 4 and 9, so $(x + 4)(x + 9)$.
5. **line 2 —** The squares are hiding behind a common factor. Both 12 and 75 are multiples of 3, and after taking it out you get $4x^2 - 25$. $12x^2 - 75 = 3(4x^2 - 25) = 3(2x - 5)(2x + 5)$.