Greatest common factor & grouping

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Count the terms. For each term, note the number and the power of each letter. Is the same number, letter or bracket in EVERY term?
2. 🎯 **Asked** The expression written as a product, factored FULLY: nothing common is left and no bracket can be factored again.
3. ✏️ **Picture it** A box: the GCF goes outside, and each term divided by the GCF goes inside, one per cell. For four terms, draw a line between the first two and the last two.
4. 🧰 **Tool** GCF = the biggest number that divides every coefficient × each letter to its LOWEST power; divide every term by it. Four terms: factor each pair so that the two brackets match (take out a negative if the third term is negative), then take the bracket out.
5. ✅ **Check** Expand your answer: it must give back every original term, signs included. Then look at each bracket: is it a difference of squares?

⭐ Must know

- GCF first: largest number dividing every coefficient × each variable to its LOWEST power.
- A bracket can be the common factor: $2x(x-3)+5(x-3)=(x-3)(2x+5)$.
- Four terms → group in pairs; opposite brackets: $(2-x)=-(x-2)$.

$$ab+ac=a(b+c)$$

$$ax+ay+bx+by=a(x+y)+b(x+y)=(x+y)(a+b)$$

🔍 Watch out for

1. Taking out a factor that is not the greatest
2. Losing the 1 left by a term equal to the GCF
3. A sign slip in the second pair
4. Stopping before a difference of squares
5. Treating opposite brackets as the same

🌱 Small wins — tick as you go

- ○ I can divide powers, expand $-3(x-4)$, divide negatives and factor x^2-x-12. (5′)
- ○ I can look at an expression and say what is common to every term, or how to pair four terms. (5′)
- ○ I can follow, then finish, a factor-by-grouping solution all the way to the end. (8′)
- ○ I can factor a four-term cubic fully on my own with at most two hints. (8′)
- ○ I can spot the five classic common-factor and grouping mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions and check each one by expanding. (10′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Simplify $\dfrac{x^5}{x^2}$.

A. x^3 B. x^7 C. $x^{2.5}$ D. $3x$

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

2. Expand $-3(x - 4)$.

A. $-3x - 12$ B. $-3x + 12$ C. $3x - 12$ D. $-3x - 4$

If you missed it — **Distributing (and the minus sign)**: Multiply EVERY term inside the bracket: $3(x - 4) = 3x - 12$. A minus in front changes every sign inside: $-(x - 5) = -x + 5$, and $2 - (x + 3) = 2 - x - 3 = -x - 1$.

1. $-2(3x - 1) = -6x + 2$ (the -1 becomes $+2$).
2. $x(x + 2) - 3(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$.

3. Evaluate $(-12) \div (-4)$.

A. -3 B. 3 C. -8 D. 48

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

4. Factor $x^2 - x - 12$.

A. $(x - 4)(x + 3)$ B. $(x + 4)(x - 3)$ C. $(x - 6)(x + 2)$ D. $(x - 12)(x + 1)$

If you missed it — **Factoring $x^2 + bx + c$ (product and sum)**: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12$ → -4 and 3 → $(x - 4)(x + 3)$. Check by expanding.

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x + 2)(x + 5)$.
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.

Answers: 1: A · 2: B · 3: B · 4: A

🔍 Common-errors sheet

1. Taking out a factor that is not the greatest

Factor $12x^3 - 18x^2$.

1 Common factor: $6x$

2 $6x(2x^2 - 3x)$

✍ Which line goes wrong first, and why?

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2. Losing the 1 left by a term equal to the GCF

Factor $8x^2y + 4xy$.

1 Common factor: $4xy$

2 $4xy(2x)$

✍ Which line goes wrong first, and why?

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3. A sign slip in the second pair

Factor $x^3 + 4x^2 - 5x - 20$.

1 $(x^3 + 4x^2) + (-5x - 20)$

2 $x^2(x + 4) - 5(x - 4)$

3 The brackets do not match, so it cannot be factored.

✍ Which line goes wrong first, and why?

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4. Stopping before a difference of squares

Factor fully: $x^3 - 2x^2 - 16x + 32$.

1 $x^2(x - 2) - 16(x - 2)$

2 $(x - 2)(x^2 - 16)$

3 Final answer: $(x - 2)(x^2 - 16)$

✍ Which line goes wrong first, and why?

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5. Treating opposite brackets as the same

Factor $5x(x - 3) + 2(3 - x)$.

1 Common bracket: $(x - 3)$

2 $(x - 3)(5x + 2)$

✍ Which line goes wrong first, and why?

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Key (fold this part back)

1. **line 1 —** Both terms have at least x^2, so x^2 (the LOWEST power) belongs in the GCF. The bracket $2x^2 - 3x$ still has a common x, which shows you stopped too early. GCF $= 6x^2$: $12x^3 - 18x^2 = 6x^2(2x - 3)$.
2. **line 2 —** $4xy \div 4xy = 1$, not 0. Dropping it leaves one term where there were two; expanding $4xy(2x)$ gives only $8x^2y$. $8x^2y + 4xy = 4xy(2x + 1)$.
3. **line 2 —** $-5 \times (-4) = +20$, not -20. Dividing -20 by -5 gives $+4$, so the bracket is $(x + 4)$. $x^2(x + 4) - 5(x + 4) = (x + 4)(x^2 - 5)$. ($x^2 - 5$ does not factor with whole numbers.)
4. **line 3 —** The grouping is right, but "fully" means checking every bracket again. $x^2 - 16$ is a difference of squares. $x^2 - 16 = (x - 4)(x + 4)$, so the answer is $(x - 2)(x - 4)(x + 4)$.
5. **line 2 —** $(3 - x)$ is not $(x - 3)$; it is $-(x - 3)$. So $+2(3 - x) = -2(x - 3)$, and the sign of the 2 changes. $5x(x - 3) - 2(x - 3) = (x - 3)(5x - 2)$. Check with $x = 0$: original $= 6$, answer $= (-3)(-2) = 6$ ✓.