Greatest common factor & grouping

提公因式与分组分解

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Count the terms. For each term, note the number and the power of each letter. Is the same number, letter or bracket in EVERY term?
   > 数一数有几项。每一项记下数字和每个字母的次数。有没有哪个数、字母或括号在「每一项」里都出现?
2. 🎯 **Asked · 求什么** The expression written as a product, factored FULLY: nothing common is left and no bracket can be factored again.
   > 把式子写成乘积，而且要「完全」分解：没有公因式剩下，也没有哪个括号还能再分解。
3. ✏️ **Picture it · 画出来** A box: the GCF goes outside, and each term divided by the GCF goes inside, one per cell. For four terms, draw a line between the first two and the last two.
   > 画一个框：公因式写在框外，每一项除以公因式的结果写在框里，一项一格。四项式就在前两项和后两项之间画一条竖线。
4. 🧰 **Tool · 用什么工具** GCF = the biggest number that divides every coefficient × each letter to its LOWEST power; divide every term by it. Four terms: factor each pair so that the two brackets match (take out a negative if the third term is negative), then take the bracket out.
   > 公因式＝能整除所有系数的最大数 × 每个字母的「最低」次幂；每一项都除以它。四项式：两组各提公因式，让两个括号相同（第三项是负的就提负号），再把括号提出来。
5. ✅ **Check · 检验** Expand your answer: it must give back every original term, signs included. Then look at each bracket: is it a difference of squares?
   > 把答案展开：必须得回原来的每一项，符号也要一样。再看每个括号：是不是平方差?

⭐ Must know

必须记住

- GCF first: largest number dividing every coefficient × each variable to its LOWEST power.
  > 先提公因式：能整除所有系数的最大数 × 每个字母的最低次幂。
- A bracket can be the common factor: $2x(x-3)+5(x-3)=(x-3)(2x+5)$.
  > 整个括号也能当公因式：$2x(x-3)+5(x-3)=(x-3)(2x+5)$。
- Four terms → group in pairs; opposite brackets: $(2-x)=-(x-2)$.
  > 四项式 → 两两分组；括号互为相反数时用 $(2-x)=-(x-2)$。

$$ab+ac=a(b+c)$$

$$ax+ay+bx+by=a(x+y)+b(x+y)=(x+y)(a+b)$$

🔍 Watch out for

当心

1. Taking out a factor that is not the greatest
   > 提出的不是最大公因式
2. Losing the 1 left by a term equal to the GCF
   > 和公因式相同的项丢了 1
3. A sign slip in the second pair
   > 第二组符号出错
4. Stopping before a difference of squares
   > 平方差没分解就停了
5. Treating opposite brackets as the same
   > 把相反的括号当成相同

🌱 Small wins — tick as you go

一小步一小步——做完打勾

- ○ I can divide powers, expand $-3(x-4)$, divide negatives and factor x^2-x-12.
  > 我会做同底幂相除、展开 $-3(x-4)$、负数相除，也会分解 x^2-x-12。
  (5′)
- ○ I can look at an expression and say what is common to every term, or how to pair four terms.
  > 我能看出一个式子里每一项共有什么，或者四项该怎么两两分组。
  (5′)
- ○ I can follow, then finish, a factor-by-grouping solution all the way to the end.
  > 我能看懂并补全一道分组分解题，一直分解到底。
  (8′)
- ○ I can factor a four-term cubic fully on my own with at most two hints.
  > 我能在最多两个提示下，独立把一个四项三次式完全分解。
  (8′)
- ○ I can spot the five classic common-factor and grouping mistakes in someone else's work.
  > 我能在别人的解答里找出提公因式和分组分解的五个经典错误。
  (8′)
- ○ I can do the Level 1–2 practice questions and check each one by expanding.
  > 我能做出 1–2 级练习题，并用展开来检验每一道。
  (10′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. Simplify $\dfrac{x^5}{x^2}$.

> 化简 $\dfrac{x^5}{x^2}$。

A. x^3 B. x^7 C. $x^{2.5}$ D. $3x$

If you missed it · 如果做错了 — **Exponent rules (product, quotient, power)**

> **指数运算法则（同底乘、除、乘方）**

: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

> 同底数：相乘→指数相加（$x^3x^4 = x^7$）；相除→指数相减（$x^7 \div x^2 = x^5$）；幂的乘方→指数相乘（$(x^3)^2 = x^6$）。积的乘方每个因数都要乘方：$(2x)^3 = 8x^3$。

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
   > $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$。
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).
   > $\frac{6x^5}{2x^2} = 3x^3$（系数相除，指数相减）。

2. Expand $-3(x - 4)$.

> 展开 $-3(x - 4)$。

A. $-3x - 12$ B. $-3x + 12$ C. $3x - 12$ D. $-3x - 4$

If you missed it · 如果做错了 — **Distributing (and the minus sign)**

> **去括号（尤其是负号）**

: Multiply EVERY term inside the bracket: $3(x - 4) = 3x - 12$. A minus in front changes every sign inside: $-(x - 5) = -x + 5$, and $2 - (x + 3) = 2 - x - 3 = -x - 1$.

> 括号里的每一项都要乘到：$3(x - 4) = 3x - 12$。前面是负号时，括号里每一项都变号：$-(x - 5) = -x + 5$，$2 - (x + 3) = 2 - x - 3 = -x - 1$。

1. $-2(3x - 1) = -6x + 2$ (the -1 becomes $+2$).
   > $-2(3x - 1) = -6x + 2$（-1 变成 $+2$）。
2. $x(x + 2) - 3(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$.
   > $x(x + 2) - 3(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$。

3. Evaluate $(-12) \div (-4)$.

> 计算 $(-12) \div (-4)$。

A. -3 B. 3 C. -8 D. 48

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

> 减去负数等于加上它：$5 - (-3) = 5 + 3 = 8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2 = 16$，但 $-4^2 = -16$（先算平方）。

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
   > $-7 - (-2)$：把「减负」改成「加」：$-7 + 2 = -5$。
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
   > $(-3)(-5) = 15$（同号），$(-3)(5) = -15$（异号）。
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.
   > 把 $x = -3$ 代入 x^2：先加括号，$(-3)^2 = 9$。

4. Factor $x^2 - x - 12$.

> 分解 $x^2 - x - 12$。

A. $(x - 4)(x + 3)$ B. $(x + 4)(x - 3)$ C. $(x - 6)(x + 2)$ D. $(x - 12)(x + 1)$

If you missed it · 如果做错了 — **Factoring $x^2 + bx + c$ (product and sum)**

> **分解 $x^2 + bx + c$（积与和）**

: Take out a common factor first. Then find two numbers that MULTIPLY to c and ADD to b: $x^2 - x - 12$ → -4 and 3 → $(x - 4)(x + 3)$. Check by expanding.

> 先提公因式。再找两个数，相乘等于 c、相加等于 b：$x^2 - x - 12$ → -4 和 3 → $(x - 4)(x + 3)$。展开检验。

1. $x^2 + 7x + 10$: $2 \times 5 = 10$, $2 + 5 = 7$ → $(x + 2)(x + 5)$.
   > $x^2 + 7x + 10$：$2 \times 5 = 10$，$2 + 5 = 7$ → $(x + 2)(x + 5)$。
2. If $c < 0$ the two numbers have different signs; if $c > 0$ they share the sign of b.
   > 若 $c < 0$，两个数异号；若 $c > 0$，两个数与 b 同号。

Answers · 答案: 1: A · 2: B · 3: B · 4: A

🔍 Common-errors sheet

常见错误单

1. Taking out a factor that is not the greatest

> **提出的不是最大公因式**

Factor $12x^3 - 18x^2$.

> 分解因式 $12x^3 - 18x^2$。

1 Common factor: $6x$

> 公因式：$6x$

2 $6x(2x^2 - 3x)$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Losing the 1 left by a term equal to the GCF

> **和公因式相同的项丢了 1**

Factor $8x^2y + 4xy$.

> 分解因式 $8x^2y + 4xy$。

1 Common factor: $4xy$

> 公因式：$4xy$

2 $4xy(2x)$

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. A sign slip in the second pair

> **第二组符号出错**

Factor $x^3 + 4x^2 - 5x - 20$.

> 分解因式 $x^3 + 4x^2 - 5x - 20$。

1 $(x^3 + 4x^2) + (-5x - 20)$

2 $x^2(x + 4) - 5(x - 4)$

3 The brackets do not match, so it cannot be factored.

> 括号不一样，所以不能分解。

✍️ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Stopping before a difference of squares

> **平方差没分解就停了**

Factor fully: $x^3 - 2x^2 - 16x + 32$.

> 完全分解：$x^3 - 2x^2 - 16x + 32$。

1 $x^2(x - 2) - 16(x - 2)$

2 $(x - 2)(x^2 - 16)$

3 Final answer: $(x - 2)(x^2 - 16)$

> 最终答案：$(x - 2)(x^2 - 16)$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Treating opposite brackets as the same

> **把相反的括号当成相同**

Factor $5x(x - 3) + 2(3 - x)$.

> 分解因式 $5x(x - 3) + 2(3 - x)$。

1 Common bracket: $(x - 3)$

> 公共括号：$(x - 3)$

2 $(x - 3)(5x + 2)$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 1 —** Both terms have at least x^2, so x^2 (the LOWEST power) belongs in the GCF. The bracket $2x^2 - 3x$ still has a common x, which shows you stopped too early.

   > 两项都至少含 x^2，所以公因式里要放 x^2（取「最低」次幂）。括号 $2x^2 - 3x$ 里还有公因式 x，说明停早了。

   GCF $= 6x^2$: $12x^3 - 18x^2 = 6x^2(2x - 3)$.

   > 公因式 $= 6x^2$：$12x^3 - 18x^2 = 6x^2(2x - 3)$。

2. **line · 第 2 —** $4xy \div 4xy = 1$, not 0. Dropping it leaves one term where there were two; expanding $4xy(2x)$ gives only $8x^2y$.

   > $4xy \div 4xy = 1$，不是 0。丢掉它就从两项变成了一项；展开 $4xy(2x)$ 只得到 $8x^2y$。

   $8x^2y + 4xy = 4xy(2x + 1)$.

   > $8x^2y + 4xy = 4xy(2x + 1)$。

3. **line · 第 2 —** $-5 \times (-4) = +20$, not -20. Dividing -20 by -5 gives $+4$, so the bracket is $(x + 4)$.

   > $-5 \times (-4) = +20$，不是 -20。$-20 \div (-5) = +4$，括号应是 $(x + 4)$。

   $x^2(x + 4) - 5(x + 4) = (x + 4)(x^2 - 5)$. ($x^2 - 5$ does not factor with whole numbers.)

   > $x^2(x + 4) - 5(x + 4) = (x + 4)(x^2 - 5)$。（$x^2 - 5$ 不能用整数再分解。）

4. **line · 第 3 —** The grouping is right, but "fully" means checking every bracket again. $x^2 - 16$ is a difference of squares.

   > 分组没错，但「完全分解」要把每个括号再检查一遍。$x^2 - 16$ 是平方差。

   $x^2 - 16 = (x - 4)(x + 4)$, so the answer is $(x - 2)(x - 4)(x + 4)$.

   > $x^2 - 16 = (x - 4)(x + 4)$，所以答案是 $(x - 2)(x - 4)(x + 4)$。

5. **line · 第 2 —** $(3 - x)$ is not $(x - 3)$; it is $-(x - 3)$. So $+2(3 - x) = -2(x - 3)$, and the sign of the 2 changes.

   > $(3 - x)$ 不是 $(x - 3)$，而是 $-(x - 3)$。所以 $+2(3 - x) = -2(x - 3)$，2 前面的符号要变。

   $5x(x - 3) - 2(x - 3) = (x - 3)(5x - 2)$. Check with $x = 0$: original $= 6$, answer $= (-3)(-2) = 6$ ✓.

   > $5x(x - 3) - 2(x - 3) = (x - 3)(5x - 2)$。用 $x = 0$ 检验：原式 $= 6$，答案 $= (-3)(-2) = 6$ ✓。