Simplifying & ordering radicals

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Read the index (2 or 3) and the radicand. With letters, note which exponents are odd and whether you are told $x \geq 0$.
2. 🎯 **Asked** Simplest form: no perfect-square factor left inside (perfect cube for $\sqrt[3]{\ }$), plus a restriction if an even root holds a letter. For ordering: a list from least to greatest.
3. ✏️ **Picture it** A factor tree of the radicand. Circle each PAIR of equal primes (each triple for a cube root); every circled group comes out as one number.
4. 🧰 **Tool** Try the perfect squares from big to small (100, 81, 64, 49, 36, 25, 16, 9, 4) until one divides the radicand; split and take its root out. To compare, square the outside number and move it in.
5. ✅ **Check** Multiply back: (outside)$^2 \times$ inside must give the original radicand. Then ask: can the leftover still be divided by 4, 9 or 25?

⭐ Must know

- Pull out the LARGEST perfect square (perfect cube for $\sqrt[3]{\ }$): $\sqrt{72} = 6\sqrt{2}$.
- Mixed → entire: square the outside number and multiply it in: $3\sqrt{5} = \sqrt{45}$.
- To compare radicals, rewrite all as entire radicals and compare the insides.

$$\sqrt{ab} = \sqrt{a}\,\sqrt{b} \quad (a, b \geq 0)$$

$$k\sqrt[n]{a} = \sqrt[n]{k^n a}$$

🔍 Watch out for

1. Stopping before the largest square is out
2. Not squaring the outside number when moving it in
3. Comparing only the outside numbers
4. Taking a whole odd power out of the root
5. Splitting a root over a plus sign

🌱 Small wins — tick as you go

- ○ I can name the perfect squares up to 144, find $\sqrt[3]{-27}$, and split a number into primes. (5′)
- ○ I can read the index and the radicand and say what "simplest form" has to look like. (5′)
- ○ I can follow, then finish, a largest-perfect-square simplification and check it by multiplying back. (8′)
- ○ I can simplify one on my own with at most two hints. (6′)
- ○ I can spot the five classic radical mistakes in someone else's work. (6′)
- ○ I can do the Level 1–2 practice questions, including putting radicals in order. (5′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Which number is a perfect square?
A. 32 B. 48 C. 64 D. 72

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.

2. What is $\sqrt[3]{-27}$?
A. -3 B. 3 C. -9 D. It has no real value

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3. Write 72 as a product of prime numbers.
A. $2^2 \cdot 3^3$ B. $2^3 \cdot 3^2$ C. $2 \cdot 36$ D. $2^4 \cdot 3$

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3 x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

4. Simplify $\sqrt{x^6}$ for $x \geq 0$.
A. x^4 B. $3x$ C. x^{36} D. x^3

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Answers: 1: C · 2: A · 3: B · 4: D

🔍 Common-errors sheet

1. Stopping before the largest square is out

Simplify $\sqrt{200}$.

1 $200 = 4 \cdot 50$, so $\sqrt{200} = 2\sqrt{50}$

2 Final answer: $2\sqrt{50}$

✍ Which line goes wrong first, and why?

2. Not squaring the outside number when moving it in

Write $5\sqrt{3}$ as an entire radical.

1 $5\sqrt{3} = \sqrt{5 \cdot 3}$

2 $= \sqrt{15}$

✍ Which line goes wrong first, and why?

3. Comparing only the outside numbers

Which is greater, $3\sqrt{7}$ or $2\sqrt{17}$?

1 Compare the outside numbers: $3 > 2$.

2 So $3\sqrt{7} > 2\sqrt{17}$.

✍ Which line goes wrong first, and why?

4. Taking a whole odd power out of the root

Simplify $\sqrt{20x^5}$ and state the restriction.

1 $20 = 4 \cdot 5$, so $\sqrt{20} = 2\sqrt{5}$

2 $\sqrt{x^5} = x^5$

3 Answer: $2x^5\sqrt{5}$

✍ Which line goes wrong first, and why?

5. Splitting a root over a plus sign

Simplify $\sqrt{36 + 64}$.

1 $\sqrt{36 + 64} = \sqrt{36} + \sqrt{64}$

2 $= 6 + 8 = 14$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** The first square that came to mind was 4, and line 1 is true. But $50 = 25 \cdot 2$ still hides a square, so the answer is not finished. Keep going: $2\sqrt{50} = 2 \cdot 5\sqrt{2} = 10\sqrt{2}$. Faster: $200 = 100 \cdot 2$.
2. **line 1 —** Moving in feels like "just put it under the root". But 5 is the root of 25, so it goes in as 25. $5\sqrt{3} = \sqrt{25 \cdot 3} = \sqrt{75}$. Check: $\sqrt{15} \approx 3.9$ but $5\sqrt{3} \approx 8.7$.
3. **line 1 —** It looks like comparing coefficients of like terms. But the radicands are different, so the outside number alone tells you nothing. Move the outside numbers in: $3\sqrt{7} = \sqrt{63}$ and $2\sqrt{17} = \sqrt{68}$. Since $68 > 63$, $2\sqrt{17}$ is greater.
4. **line 2 —** The letter was treated as if it were already a square. A square root HALVES an exponent, and only an even power can come out: $x^5 = x^4 \cdot x$. $20x^5 = (4x^4)(5x)$, so $\sqrt{20x^5} = 2x^2\sqrt{5x}$, with $x \geq 0$.
5. **line 1 —** The product rule $\sqrt{ab} = \sqrt{a}\sqrt{b}$ is real, so it is tempting to use the same move for $+$. There is no such rule for addition. Add first: $\sqrt{100} = 10$. (Check: $14^2 = 196 \neq 100$.)