Rational exponents

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Circle three things: the base, the denominator of the exponent (the root) and the numerator (the power). Note any minus sign in the exponent.
2. 🎯 **Asked** Is it "evaluate" (a number), "simplify" (one power of each variable, positive exponents) or "write as a radical"? The answer form depends on it.
3. ✏️ **Picture it** Write the base as a power of a small number: $64 = 2^6 = 4^3 = 8^2$. Seeing that list tells you which root comes out clean.
4. 🧰 **Tool** Flip first if the exponent is negative, then take the root, then the power. For variables, use the exponent laws and add/subtract the fractions.
5. ✅ **Check** A negative exponent never makes the answer negative. Check backwards: raise your answer to the reciprocal power and see if you get the start again.

⭐ Must know

- $a^{m/n} = (\sqrt[n]{a})^m$: bottom of the exponent = root, top = power.
- By hand, take the root first: $27^{2/3} = 3^2 = 9$.
- Negative exponent = reciprocal, never a negative answer: $16^{-1/2} = \frac{1}{4}$.

$$a^{\frac{1}{n}} = \sqrt[n]{a}, \qquad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}$$

🔍 Watch out for

1. Reading the root and the power the wrong way round
2. A negative exponent gives a negative answer
3. Multiplying the base by the exponent
4. Multiplying exponents when you should add them
5. Raising only the variable, not the number in front

🌱 Small wins — tick as you go

- ○ I can use $x^a \cdot x^b = x^{a+b}$, find small cube roots, and add fractions like $\frac{2}{3} + \frac{1}{4}$. (5′)
- ○ I can read $a^{m/n}$ out loud as "the nth root, then the mth power" and say what a minus sign in the exponent does. (5′)
- ○ I can follow, then finish, the flip–root–power method for $a^{-m/n}$. (8′)
- ○ I can evaluate a negative rational power of a fraction on my own with at most two hints. (6′)
- ○ I can spot the five classic rational-exponent mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions without a calculator. (8′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Simplify $x^3 \cdot x^4$.
A. x^{12} B. x^7 C. $2x^7$ D. x^{34}

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

2. What is $\sqrt[3]{64}$?
A. 4 B. 8 C. $\frac{64}{3}$ D. 16

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.

3. Work out $\frac{2}{3} + \frac{1}{4}$.
A. $\frac{3}{7}$ B. $\frac{2}{12}$ C. $\frac{5}{12}$ D. $\frac{11}{12}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.

4. What is 2^{-3}?
A. -8 B. -6 C. $\frac{1}{8}$ D. $\frac{1}{6}$

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

Answers: 1: B · 2: A · 3: D · 4: C

🔍 Common-errors sheet

1. Reading the root and the power the wrong way round

Evaluate $64^{2/3}$.

1 Square root of 64: $\sqrt{64} = 8$

2 Cube it: $8^3 = 512$

✍ Which line goes wrong first, and why?

.....

2. A negative exponent gives a negative answer

Evaluate $25^{-1/2}$.

1 $\sqrt{25} = 5$

2 The exponent is negative, so the answer is -5.

✍ Which line goes wrong first, and why?

.....

3. Multiplying the base by the exponent

Evaluate $27^{1/3}$.

1 $27^{1/3} = 27 \times \frac{1}{3}$

2 $= 9$

✍ Which line goes wrong first, and why?

.....

4. Multiplying exponents when you should add them

Simplify $x^{1/2} \cdot x^{1/4}$, $x > 0$.

1 Same base, so $x^{\frac{1}{2} \cdot \frac{1}{4}}$

2 $= x^{1/8}$

✍ Which line goes wrong first, and why?

.....

5. Raising only the variable, not the number in front

Simplify $(16x^8)^{3/4}$, $x > 0$.

1 $x^{8 \cdot \frac{3}{4}} = x^6$

2 So $(16x^8)^{3/4} = 16x^6$

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 1** — The 2 on top looked like "square root". But the DENOMINATOR is the root: $\frac{2}{3}$ means cube root, then square. $\sqrt[3]{64} = 4$, then $4^2 = 16$.
2. **line 2** — The minus sign in an exponent means "take the reciprocal", not "make it negative". A positive base always gives a positive power. $25^{-1/2} = \dfrac{1}{25^{1/2}} = \dfrac{1}{5}$.
3. **line 1** — An exponent is not a multiplier. $\frac{1}{3}$ as an exponent asks "what number cubed gives 27?" $27^{1/3} = \sqrt[3]{27} = 3$, since $3^3 = 27$.
4. **line 1** — Fractions made the familiar rule feel new. Multiplying powers of the same base still means ADD the exponents; you multiply only for a power of a power. $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$, so $x^{3/4}$.
5. **line 2** — The 16 looked like a plain coefficient that just comes along. But the bracket means every factor inside is raised to $\frac{3}{4}$. $16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$, so $8x^6$.