The real number system

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Write each number exactly as given. Note what it has: a root (which root, and of what), a fraction, a repeating bar, π.
2. 🎯 **Asked** Which sets does it belong to? List ALL of them from $\mathbb{N}$, $\mathbb{W}$, $\mathbb{I}$, $\mathbb{Q}$, $\overline{\mathbb{Q}}$, $\mathbb{R}$, or say "not real".
3. ✏️ **Picture it** Nested boxes: $\mathbb{N}$ inside $\mathbb{W}$ inside $\mathbb{I}$ inside $\mathbb{Q}$, with $\overline{\mathbb{Q}}$ beside $\mathbb{Q}$, all inside $\mathbb{R}$. Find the smallest box the number fits; it is also in every box around that one.
4. 🧰 **Tool** Simplify first: work out roots of perfect squares and cubes, reduce fractions, turn repeating decimals into fractions. An even root of a negative is not real, so stop there.
5. ✅ **Check** Decimal test: it ends or repeats, so rational; it never ends and never repeats, so irrational. Then make sure you listed every bigger set too.

⭐ Must know

- $\mathbb{N} \subset \mathbb{W} \subset \mathbb{I} \subset \mathbb{Q} \subset \mathbb{R}$; 0 is whole but not natural.
- Rational = $\frac{a}{b}$ with integers a, b: decimal ends or repeats. Irrational: never ends, never repeats.
- Simplify before classifying: $\sqrt{49} = 7$, $\sqrt[3]{-8} = -2$; $\sqrt{-4}$ is not real.

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{I} \subset \mathbb{Q} \subset \mathbb{R}, \qquad \mathbb{R} = \mathbb{Q} \cup \overline{\mathbb{Q}}$$

$$0.\overline{ab} = \frac{ab}{99}$$

🔍 Watch out for

1. Listing only the smallest set
2. Calling a number irrational because of the root sign
3. Thinking any fraction shape is rational
4. Turning an even root of a negative into a negative number
5. Deciding from the calculator screen

🌱 Small wins — tick as you go

- ○ I can spot perfect squares and cubes and reduce a fraction. (5′)
- ○ I can ask the five starting questions before I classify a number. (5′)
- ○ I can follow, then finish, a simplify–test–climb classification. (8′)
- ○ I can classify three new numbers on my own with at most two hints. (6′)
- ○ I can spot the five classic classification mistakes in someone else's work. (8′)
- ○ I can do the Level 1–2 practice questions. (8′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. Which number is a perfect square?

A. 50 B. 64 C. 80 D. 90

If you missed it — **Perfect squares and square roots**: Know the squares $1, 4, 9, \ldots, 225$. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. But $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$: $\sqrt{9+16} = 5$, not 7. $\sqrt{x}$ means the non-negative root.

1. $\sqrt{72}$: biggest square factor is 36 → $6\sqrt{2}$.
2. $x^2 = 49$ has TWO answers $x = \pm 7$, but $\sqrt{49} = 7$ only.

2. Work out $(-3)^3$.

A. -27 B. 27 C. -9 D. 9

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

3. Write $\dfrac{36}{99}$ in lowest terms.

A. $\dfrac{2}{9}$ B. $\dfrac{4}{11}$ C. $\dfrac{9}{11}$ D. $\dfrac{6}{33}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.

4. Write 0.75 as a fraction in lowest terms.

A. $\dfrac{3}{5}$ B. $\dfrac{7}{5}$ C. $\dfrac{3}{4}$ D. $\dfrac{75}{10}$

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Answers: 1: B · 2: A · 3: B · 4: C

🔍 Common-errors sheet

1. Listing only the smallest set

Name every set that $\sqrt{81}$ belongs to.

1 $\sqrt{81} = 9$.

2 9 is a counting number, so the answer is $\mathbb{N}$.

✍ Which line goes wrong first, and why?

2. Calling a number irrational because of the root sign

Is $\sqrt{0.09}$ rational or irrational?

1 It has a root sign, and 0.09 is not a perfect square like 4 or 9.

2 So $\sqrt{0.09}$ is irrational.

✍ Which line goes wrong first, and why?

3. Thinking any fraction shape is rational

Is $\dfrac{\sqrt{3}}{2}$ rational or irrational?

1 It is written as a top over a bottom.

2 A fraction is rational, so $\dfrac{\sqrt{3}}{2}$ is rational.

✍ Which line goes wrong first, and why?

4. Turning an even root of a negative into a negative number

Classify $\sqrt{-16}$.

1 $\sqrt{-16} = -4$.

2 -4 is an integer, so $\mathbb{I}, \mathbb{Q}, \mathbb{R}$.

✍ Which line goes wrong first, and why?

5. Deciding from the calculator screen

Is $\frac{7}{13}$ rational or irrational?

1 The calculator shows 0.5384615385.

2 No repeat shows on the screen, so it is irrational.

✍ Which line goes wrong first, and why?

Key (fold this part back)

1. **line 2 —** Finding the first set that fits feels like the end of the job. But the sets are nested, so 9 also sits in every bigger set. Climb the ladder: $9 \in \mathbb{N}, \mathbb{W}, \mathbb{I}, \mathbb{Q}, \mathbb{R}$.
2. **line 1 —** Only whole numbers were tried as perfect squares. Decimals can be perfect squares too: $0.3^2 = 0.09$. $\sqrt{0.09} = 0.3 = \frac{3}{10}$, a ratio of integers, so it is rational.
3. **line 2 —** Rational means a ratio of two INTEGERS. The shape matches, but the top, $\sqrt{3}$, is not an integer. $\sqrt{3}$ is irrational, and half of an irrational number is still irrational.
4. **line 1 —** It copies the cube-root habit ($\sqrt[3]{-8} = -2$). But $(-4)^2 = 16$, not -16: no real number squares to a negative. Stop at line 1: $\sqrt{-16}$ is not a real number, so it is in none of the sets.
5. **line 2 —** The screen only has room for about ten digits. The block 538461 has six digits, so its repeat is cut off by rounding. Decide from the exact form: $\frac{7}{13}$ is a ratio of integers, so it is rational ($0.\overline{538461}$).