Exponent laws (zero & negative)

Thinking map — keep it next to you while you work

🧭 Five questions before you write

1. 🔎 **Given** Circle every base and its exponent. Mark each bracket and the power outside it, and every negative or zero exponent.
2. 🎯 **Asked** One tidy expression: each base written once, the numbers multiplied out, and positive exponents only (or just a number).
3. ✏️ **Picture it** Draw an arrow from the outside power to EVERY factor in the bracket, the number included. If a rule feels shaky, write it out once: $x^3 \cdot x^2 = xxx \cdot xx$.
4. 🧰 **Tool** Brackets first (multiply exponents, power the number), then the numbers, then one base at a time: add for $\times$, subtract for $\div$. Last, move negative exponents across the fraction bar.
5. ✅ **Check** Put a small number such as $x = 2$, $y = 1$ into the original and into your answer; both must give the same value. Then look: no negative exponent left.

⭐ Must know

- Same base: multiply → add exponents, divide → subtract, power of a power → multiply.
- An outside power hits every factor, numbers too: $(3x^2)^3 = 27x^6$.
- $a^0 = 1$; $a^{-n} = \frac{1}{a^n}$ — a negative exponent means reciprocal, never a negative number.

$$a^m \cdot a^n = a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n}, \quad (a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

🔍 Watch out for

1. Not raising the number in the bracket
2. Reading a negative exponent as a negative number
3. Adding exponents for a power of a power
4. Subtracting a negative exponent the wrong way
5. Giving the zero power to the wrong part

🌱 Small wins — tick as you go

- ○ I can work out small powers like 3^4 and $(-2)^3$, and add and divide simple fractions. (5′)
- ○ I can circle every base, see what an outside power touches, and say what the final answer must look like. (5′)
- ○ I can follow, then finish, a brackets–numbers–bases–positive solution of one big fraction. (8′)
- ○ I can simplify one on my own with at most two hints and check it by putting in numbers. (7′)
- ○ I can spot the five classic exponent mistakes in someone else's work. (5′)
- ○ I can do the Level 1–2 practice questions with positive exponents in every answer. (5′)

Prerequisite repair sheet

Do the question first. Missed it? Read the rule and the worked lines under it, then try again.

1. What is 3^4?
A. 12 B. 7 C. 81 D. 64

If you missed it — **Exponent rules (product, quotient, power)**: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).

2. Evaluate $(-2)^3$.
A. -8 B. 8 C. -6 D. 6

If you missed it — **Negative numbers: add, subtract, multiply**: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.

3. Work out $\frac{1}{4} + \frac{1}{8}$.
A. $\frac{2}{12}$ B. $\frac{1}{6}$ C. $\frac{2}{8}$ D. $\frac{3}{8}$

If you missed it — **Fractions: add, subtract, multiply, divide**: Add or subtract only with a common denominator: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

1. $\frac{2}{3} - \frac{1}{4}$: common denominator 12 → $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.
2. $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$.
3. Never add denominators: $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.

4. Work out $\frac{2}{3} \div \frac{1}{9}$.
A. $\frac{2}{27}$ B. 6 C. $\frac{3}{2}$ D. 18

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Answers: 1: C · 2: A · 3: D · 4: B

🔍 Common-errors sheet

1. Not raising the number in the bracket

Simplify $(5x^3)^2$.

1 $(5x^3)^2 = 5 \cdot x^{3 \cdot 2}$

2 $= 5x^6$

✍ Which line goes wrong first, and why?

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2. Reading a negative exponent as a negative number

Evaluate $3^{-2} + 3^0$.

1 $3^{-2} = -9$ and $3^0 = 1$

2 $-9 + 1 = -8$

✍ Which line goes wrong first, and why?

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3. Adding exponents for a power of a power

Simplify $(x^{-2})^3 \cdot x^4$ with positive exponents.

1 $(x^{-2})^3 = x^{-2+3} = x^1$

2 $x \cdot x^4 = x^5$

✍ Which line goes wrong first, and why?

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4. Subtracting a negative exponent the wrong way

Simplify $\dfrac{6a^4}{2a^{-3}}$.

1 Numbers: $\frac{6}{2} = 3$

2 $a^{4-3} = a^1$

3 Answer: $3a$

✍ Which line goes wrong first, and why?

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5. Giving the zero power to the wrong part

Simplify $4x^0 + (4x)^0$.

1 $4x^0 = 1$

2 $(4x)^0 = 1$

3 $1 + 1 = 2$

✍ Which line goes wrong first, and why?

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Key (fold this part back)

1. **line 1 —** The eye goes to the letter, so the power is only given to x^3. But the outside 2 squares the WHOLE bracket, and 5 is part of it. $(5x^3)^2 = 5^2 \cdot x^6 = 25x^6$.
2. **line 1 —** A minus sign usually means "negative", so it feels natural here too. But a negative EXPONENT means "one over": $3^{-2} = \dfrac{1}{3^2}$. $3^{-2} = \frac{1}{9}$, so $\frac{1}{9} + 1 = \frac{10}{9}$.
3. **line 1 —** The two rules got swapped. "Add" is for $x^a \cdot x^b$; a power OF a power means the factor is repeated 3 times, so the exponents multiply: $(-2)(3)$. $(x^{-2})^3 = x^{-6}$, then $x^{-6} \cdot x^4 = x^{-2} = \dfrac{1}{x^2}$.
4. **line 2 —** The rule is "top exponent minus bottom exponent", and the bottom one is -3. Dropping the brackets turns $4 - (-3)$ into $4 - 3$. $a^{4-(-3)} = a^7$, so the answer is $3a^7$. (Or move a^{-3} up as a^3: $a^4 \cdot a^3 = a^7$.)
5. **line 1 —** "Anything to the power 0 is 1" was applied to the whole term. In $4x^0$ the 0 belongs only to x; there is no bracket, so the 4 stays. $4x^0 = 4 \cdot 1 = 4$ and $(4x)^0 = 1$, so $4 + 1 = 5$.