Exponent laws (zero & negative)

指数律（零指数与负指数）

Thinking map — keep it next to you while you work · 思路图——做题时放在手边

🧭 Five questions before you write

动笔前的五个问题

1. 🔎 **Given · 已知** Circle every base and its exponent. Mark each bracket and the power outside it, and every negative or zero exponent.
   > 圈出每个底数和它的指数。标出每个括号以及括号外的指数，还有所有负指数、零指数。
2. 🎯 **Asked · 求什么** One tidy expression: each base written once, the numbers multiplied out, and positive exponents only (or just a number).
   > 一个整洁的式子：每个底数只出现一次，数字算出来，只用正指数（或者就是一个数）。
3. ✏️ **Picture it · 画出来** Draw an arrow from the outside power to EVERY factor in the bracket, the number included. If a rule feels shaky, write it out once: $x^3 \cdot x^2 = xxx \cdot xx$.
   > 从括号外的指数画箭头指向括号里的「每一个」因数，数字也算。哪条法则没把握，就展开写一次：$x^3 \cdot x^2 = xxx \cdot xx$。
4. 🧰 **Tool · 用什么工具** Brackets first (multiply exponents, power the number), then the numbers, then one base at a time: add for $\times$, subtract for $\div$. Last, move negative exponents across the fraction bar.
   > 先去括号（指数相乘，数字也乘方），再算数字，然后一个底数一个底数地处理：相乘指数相加，相除指数相减。最后把负指数移到分数线另一边。
5. ✅ **Check · 检验** Put a small number such as $x = 2$, $y = 1$ into the original and into your answer; both must give the same value. Then look: no negative exponent left.
   > 把 $x = 2$、$y = 1$ 这样的小数代入原式和你的答案，两者的值必须相同。再看一眼：不能留负指数。

⭐ Must know

必须记住

- Same base: multiply → add exponents, divide → subtract, power of a power → multiply.
  > 同底数：相乘指数相加，相除指数相减，乘方的乘方指数相乘。
- An outside power hits every factor, numbers too: $(3x^2)^3 = 27x^6$.
  > 括号外的指数作用到每个因式上，系数也要乘方：$(3x^2)^3 = 27x^6$。
- $a^0 = 1$; $a^{-n} = \frac{1}{a^n}$ — a negative exponent means reciprocal, never a negative number.
  > $a^0 = 1$；$a^{-n} = \frac{1}{a^n}$——负指数表示倒数，不是负数。

$$a^m \cdot a^n = a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n}, \quad (a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

🔍 Watch out for

当心

1. Not raising the number in the bracket
   > 括号里的数字没有乘方
2. Reading a negative exponent as a negative number
   > 把负指数当成负数
3. Adding exponents for a power of a power
   > 幂的乘方把指数相加了
4. Subtracting a negative exponent the wrong way
   > 减去负指数时符号错了
5. Giving the zero power to the wrong part
   > 零指数作用错了对象

🌱 Small wins — tick as you go

一小步一小步——做完打勾

- ○ I can work out small powers like 3^4 and $(-2)^3$, and add and divide simple fractions.
  > 我会算 3^4、$(-2)^3$ 这样的小乘方，也会做简单分数的加法和除法。

  (5′)
- ○ I can circle every base, see what an outside power touches, and say what the final answer must look like.
  > 我能圈出每个底数，看清括号外的指数作用到谁，并说出最终答案应该长什么样。

  (5′)
- ○ I can follow, then finish, a brackets–numbers–bases–positive solution of one big fraction.
  > 我能看懂并补全一道大分式的「去括号—算数字—逐个底数—变正指数」解法。

  (8′)
- ○ I can simplify one on my own with at most two hints and check it by putting in numbers.
  > 我能在最多两个提示下独立化简一道，并代数字检验。

  (7′)
- ○ I can spot the five classic exponent mistakes in someone else's work.
  > 我能在别人的解答里找出指数运算的五个经典错误。

  (5′)
- ○ I can do the Level 1–2 practice questions with positive exponents in every answer.
  > 我能做出 1–2 级练习题，而且每个答案都只写正指数。

  (5′)

Prerequisite repair sheet

先修补救单

Do the question first. Missed it? Read the rule and the worked lines under it, then try again. · 先做题。做错了？读下面的规则和例子，再做一遍。

1. What is 3^4?

> 3^4 等于多少?

A. 12 B. 7 C. 81 D. 64

If you missed it · 如果做错了 — **Exponent rules (product, quotient, power)**

> **指数运算法则（同底乘、除、乘方）**

: Same base: multiply → ADD exponents ($x^3x^4 = x^7$); divide → SUBTRACT ($x^7 \div x^2 = x^5$); power of a power → MULTIPLY ($(x^3)^2 = x^6$). A power of a product hits every factor: $(2x)^3 = 8x^3$.

> 同底数：相乘→指数相加（$x^3x^4 = x^7$）；相除→指数相减（$x^7 \div x^2 = x^5$）；幂的乘方→指数相乘（$(x^3)^2 = x^6$）。积的乘方每个因数都要乘方：$(2x)^3 = 8x^3$。

1. $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$.
   > $(3x^2)^2 \cdot x = 9x^4 \cdot x = 9x^5$。
2. $\frac{6x^5}{2x^2} = 3x^3$ (divide the numbers, subtract the exponents).
   > $\frac{6x^5}{2x^2} = 3x^3$（系数相除，指数相减）。

2. Evaluate $(-2)^3$.

> 计算 $(-2)^3$。

A. -8 B. 8 C. -6 D. 6

If you missed it · 如果做错了 — **Negative numbers: add, subtract, multiply**

> **负数的加减乘**

: Subtracting a negative is adding: $5 - (-3) = 5 + 3 = 8$. Same signs multiply to $+$, different signs to $-$. Squaring a negative is positive: $(-4)^2 = 16$, but $-4^2 = -16$ (the square happens first).

> 减去负数等于加上它：$5 - (-3) = 5 + 3 = 8$。同号相乘得正，异号相乘得负。负数的平方是正的：$(-4)^2 = 16$，但 $-4^2 = -16$（先算平方）。

1. $-7 - (-2)$: change "minus negative" to "plus": $-7 + 2 = -5$.
   > $-7 - (-2)$：把「减负」改成「加」：$-7 + 2 = -5$。
2. $(-3)(-5) = 15$ (same signs), $(-3)(5) = -15$ (different signs).
   > $(-3)(-5) = 15$（同号），$(-3)(5) = -15$（异号）。
3. Plugging in $x = -3$ to x^2: write brackets first, $(-3)^2 = 9$.
   > 把 $x = -3$ 代入 x^2：先加括号，$(-3)^2 = 9$。

3. Work out $\frac{1}{4}+\frac{1}{8}$.

> 计算 $\frac{1}{4}+\frac{1}{8}$。

A. $\frac{2}{12}$ B. $\frac{1}{6}$ C. $\frac{2}{8}$ D. $\frac{3}{8}$

If you missed it · 如果做错了 — **Fractions: add, subtract, multiply, divide**

> **分数的四则运算**

: Add or subtract only with a common denominator: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

> 加减必须先通分：$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$。乘法分子乘分子、分母乘分母。除以一个分数＝乘它的倒数（把第二个分数倒过来）。

1. $\frac{2}{3}-\frac{1}{4}$: common denominator 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$.
   > $\frac{2}{3}-\frac{1}{4}$：公分母 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$。
2. $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$.
   > $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$。
3. Never add denominators: $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$.
   > 分母不能相加：$\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$。

4. Work out $\frac{2}{3}\div\frac{1}{9}$.

> 计算 $\frac{2}{3}\div\frac{1}{9}$。

A. $\frac{2}{27}$ B. 6 C. $\frac{3}{2}$ D. 18

If you missed it · 如果做错了 — **Fractions: add, subtract, multiply, divide**

> **分数的四则运算**

: Add or subtract only with a common denominator: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. Multiply straight across. Divide = multiply by the reciprocal (flip the second fraction).

> 加减必须先通分：$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$。乘法分子乘分子、分母乘分母。除以一个分数＝乘它的倒数（把第二个分数倒过来）。

1. $\frac{2}{3}-\frac{1}{4}$: common denominator 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$.
   > $\frac{2}{3}-\frac{1}{4}$：公分母 12 → $\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$。
2. $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$.
   > $\frac{3}{4}\div\frac{9}{8}=\frac{3}{4}\cdot\frac{8}{9}=\frac{24}{36}=\frac{2}{3}$。
3. Never add denominators: $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$.
   > 分母不能相加：$\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$。

Answers · 答案: 1: C · 2: A · 3: D · 4: B

🔍 Common-errors sheet

常见错误单

1. Not raising the number in the bracket

> **括号里的数字没有乘方**

Simplify $(5x^3)^2$.

> 化简 $(5x^3)^2$。

1 $(5x^3)^2 = 5 \cdot x^{3 \cdot 2}$

2 $= 5x^6$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

2. Reading a negative exponent as a negative number

> **把负指数当成负数**

Evaluate $3^{-2} + 3^0$.

> 计算 $3^{-2} + 3^0$。

1 $3^{-2} = -9$ and $3^0 = 1$

> $3^{-2} = -9,\ 3^0 = 1$

2 $-9 + 1 = -8$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

3. Adding exponents for a power of a power

> **幂的乘方把指数相加了**

Simplify $(x^{-2})^3 \cdot x^4$ with positive exponents.

> 化简 $(x^{-2})^3 \cdot x^4$，用正指数表示。

1 $(x^{-2})^3 = x^{-2+3} = x^1$

2 $x \cdot x^4 = x^5$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

4. Subtracting a negative exponent the wrong way

> **减去负指数时符号错了**

Simplify $\dfrac{6a^4}{2a^{-3}}$.

> 化简 $\dfrac{6a^4}{2a^{-3}}$。

1 Numbers: $\frac{6}{2} = 3$

> 数字：$\frac{6}{2} = 3$

2 $a^{4-3} = a^1$

3 Answer: $3a$

> 答案：$3a$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

5. Giving the zero power to the wrong part

> **零指数作用错了对象**

Simplify $4x^0 + (4x)^0$.

> 化简 $4x^0 + (4x)^0$。

1 $4x^0 = 1$

2 $(4x)^0 = 1$

3 $1 + 1 = 2$

✍ Which line goes wrong first, and why? · 哪一行最先出错？为什么？

Key (fold this part back) · 答案（可以折起来）

1. **line · 第 1 —** The eye goes to the letter, so the power is only given to x^3. But the outside 2 squares the WHOLE bracket, and 5 is part of it.

   > 眼睛只盯着字母，于是指数只给了 x^3。可是外面的 2 是把「整个」括号平方，5 也在括号里。

   $(5x^3)^2 = 5^2 \cdot x^6 = 25x^6$.

   > $(5x^3)^2 = 5^2 \cdot x^6 = 25x^6$。

2. **line · 第 1 —** A minus sign usually means "negative", so it feels natural here too. But a negative EXPONENT means "one over": $3^{-2} = \dfrac{1}{3^2}$.

   > 减号平时表示「负数」，所以这里也顺手当成负数。可是负「指数」的意思是「取倒数」：$3^{-2} = \dfrac{1}{3^2}$。

   $3^{-2} = \frac{1}{9}$, so $\frac{1}{9} + 1 = \frac{10}{9}$.

   > $3^{-2} = \frac{1}{9}$，所以 $\frac{1}{9} + 1 = \frac{10}{9}$。

3. **line · 第 1 —** The two rules got swapped. "Add" is for $x^a \cdot x^b$; a power OF a power means the factor is repeated 3 times, so the exponents multiply: $(-2)(3)$.

   > 两条法则弄混了。「相加」用于 $x^a \cdot x^b$；幂的乘方是把因数重复 3 次，所以指数相乘：$(-2)(3)$。

   $(x^{-2})^3 = x^{-6}$, then $x^{-6} \cdot x^4 = x^{-2} = \dfrac{1}{x^2}$.

   > $(x^{-2})^3 = x^{-6}$，再 $x^{-6} \cdot x^4 = x^{-2} = \dfrac{1}{x^2}$。

4. **line · 第 2 —** The rule is "top exponent minus bottom exponent", and the bottom one is -3. Dropping the brackets turns $4 - (-3)$ into $4 - 3$.

   > 法则是「上面的指数减下面的指数」，而下面的是 -3。不写括号，$4 - (-3)$ 就变成了 $4 - 3$。

   $a^{4-(-3)} = a^7$, so the answer is $3a^7$. (Or move a^{-3} up as a^3: $a^4 \cdot a^3 = a^7$.)

   > $a^{4-(-3)} = a^7$，答案是 $3a^7$。(也可以把 a^{-3} 移到上面变成 a^3：$a^4 \cdot a^3 = a^7$。)

5. **line · 第 1 —** "Anything to the power 0 is 1" was applied to the whole term. In $4x^0$ the 0 belongs only to x; there is no bracket, so the 4 stays.

   > 把「任何数的 0 次方都是 1」用到了整个项上。$4x^0$ 里的 0 只属于 x；没有括号，4 不变。

   $4x^0 = 4 \cdot 1 = 4$ and $(4x)^0 = 1$, so $4 + 1 = 5$.

   > $4x^0 = 4 \cdot 1 = 4$，$(4x)^0 = 1$，所以 $4 + 1 = 5$。