🎯 Solving by factoring & square roots

因式分解法与直接开平方法

Date · 日期 _____ Topic · 知识点 1/5

CUES · QUESTIONS
提示 · 问题

1. Zero product property?
   > 零积性质?
2. Solve $(x+1)^2 = 7$.
   > 解 $(x+1)^2 = 7$。
3. Solve $x^2 = -9$.
   > 解 $x^2 = -9$。
4. First move for $5x^2 = 20x$?
   > $5x^2 = 20x$ 第一步?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

NOTES
笔记

1 · Worked example · 1 · 看一道例题 Copy one example from the page. Next to each line write WHY in a few words. · 抄一道例题，每一行旁边用几个字写「为什么」。

2 · Fill in the blanks · 2 · 填空题 Cover the steps and redo it. Leave a gap where you are unsure. · 盖住步骤自己重做，不确定的地方先空着。

3 · Your turn · 3 · 自己做 A new question. Then check it against the answer. · 做一道新题，然后对答案。

SUMMARY — IN MY OWN WORDS, TWO SENTENCES
小结——用自己的话，两句

🎯 Completing the square

配方法

Date · 日期 _____ Topic · 知识点 2/5

CUES · QUESTIONS

提示 · 问题

1. What do you add to both sides of $x^2 - 10x = 4$?
   > $x^2 - 10x = 4$ 两边要加多少?
2. First step for $4x^2 + 8x - 1 = 0$?
   > $4x^2 + 8x - 1 = 0$ 第一步?
3. $(x + 2)^2 = -5$. Solutions?
   > $(x + 2)^2 = -5$ 的解?
4. Half of $b = 7$, squared?
   > $b = 7$ 时一半的平方?

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

🎯 The quadratic formula

求根公式

Date · 日期 _____ Topic · 知识点 3/5

CUES · QUESTIONS

提示 · 问题

1. Quadratic formula?
   > 求根公式?
2. First move before using the formula?
   > 套公式前第一步?
3. When is completing the square faster?
   > 什么时候配方更快?
4. Simplify $\frac{4 \pm 2\sqrt{3}}{2}$
   > 化简 $\frac{4 \pm 2\sqrt{3}}{2}$

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

🎯 The discriminant

判别式

Date · 日期 _____ Topic · 知识点 4/5

CUES · QUESTIONS

提示 · 问题

1. Discriminant formula?
   > 判别式公式?
2. $D > 0$, $D = 0$, $D < 0$ mean?
   > $D > 0$、$D = 0$、$D < 0$ 分别表示?
3. $D = 49$. What else do you know?
   > $D = 49$，还能知道什么?
4. Graph meaning of $D = 0$?
   > $D = 0$ 在图象上表示什么?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

🎯 Choosing a method & word problems

选方法与应用题

Date · 日期 _____ Topic · 知识点 5/5

CUES · QUESTIONS

提示 · 问题

1. Best method for $(x-1)^2=12$?
   > $(x-1)^2=12$ 最好用什么方法?
2. Best method for $x^2-x-12=0$?
   > $x^2-x-12=0$ 最好用什么方法?
3. Best method for $3x^2-7x+1=0$?
   > $3x^2-7x+1=0$ 最好用什么方法?
4. Consecutive odd integers?
   > 连续奇数怎么设?

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

🎯 Quadratic Equations

一元二次方程

- **Formula and trap sheet**

公式与陷阱一页纸

Everything you need before a test, on the fewest pages. Fold it and keep it in your binder. · 考前要看的东西，压缩在尽量少的页数里。折起来放进文件夹。

Solving by factoring & square roots · 因式分解法与直接开平方法

FORMULAS · 公式

$AB = 0 \;\Rightarrow\; A = 0$ or $B = 0$ — Zero product property

> 零积性质

$(x - p)^2 = k \;\Rightarrow\; x = p \pm \sqrt{k} \quad (k \geq 0)$ — Square root method

> 直接开平方法

REMEMBER · 记住

- Zero product: get 0 on one side FIRST, then set each factor $= 0$.
  > 零积法则：先让一边等于 0，再令每个因式 $= 0$。
- $(\ldots)^2 = k$: take $\pm\sqrt{k}$; $k < 0$ → no real solution.
  > $(\ldots)^2 = k$：开方取 $\pm\sqrt{k}$；$k < 0$ → 无实数解。

WATCH OUT · 当心

- $x(x - 3) = 10 \Rightarrow x = 10$ or $x - 3 = 10$ → expand: $x^2 - 3x - 10 = 0$, $x = 5, -2$.
  > $x(x - 3) = 10 \Rightarrow x = 10$ 或 $x - 3 = 10$ → 先展开：$x^2 - 3x - 10 = 0$，$x = 5, -2$。
- $x^2 = 49 \Rightarrow x = 7$ → $x = \pm 7$.
  > $x^2 = 49 \Rightarrow x = 7$ → 应为 $x = \pm 7$。

Completing the square · 配方法

FORMULAS · 公式

$x^2 + bx = k \;\Rightarrow\; \left(x + \frac{b}{2}\right)^2 = k + \left(\frac{b}{2}\right)^2$ — Completing the square in an equation

> 方程中的配方

REMEMBER · 记住

- Make $a = 1$ → move c → add $\left(\frac{b}{2}\right)^2$ to BOTH sides → take $\pm\sqrt{\ }$.
  > 化 $a = 1$ → 移常数 → 两边都加 $\left(\frac{b}{2}\right)^2$ → 开方取 $\pm$。
- Equation: add to both sides. Vertex form of a function: add AND subtract on one side.
  > 解方程：两边同加。化函数顶点式：同一边加了再减。

WATCH OUT · 当心

- $x^2 + 6x = 2$: $x^2 + 6x + 9 = 2$ → add to both sides: $= 11$.
  > $x^2 + 6x = 2$：写成 $x^2 + 6x + 9 = 2$ → 两边都加：$= 11$。
- $2x^2 + 8x = 3$: add 16 → divide by 2 first, then add 4 to both sides.
  > $2x^2 + 8x = 3$：加 16 → 先除以 2，再两边加 4。

The quadratic formula · 求根公式

FORMULAS · 公式

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic formula (for $ax^2 + bx + c = 0$, $a \neq 0$)

> 求根公式（适用于 $ax^2 + bx + c = 0$，$a \neq 0$）

REMEMBER · 记住

- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, only after the equation is $ax^2 + bx + c = 0$.
  > $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$，先化成 $ax^2 + bx + c = 0$ 再用。
- Read a, b, c with their signs; b^2 is never negative.
  > a, b, c 连符号一起读；b^2 永远不是负数。

WATCH OUT · 当心

- $\dfrac{4 \pm 2\sqrt{3}}{2} = 4 \pm \sqrt{3}$ → $2 \pm \sqrt{3}$.
  > $\dfrac{4 \pm 2\sqrt{3}}{2} = 4 \pm \sqrt{3}$ → 应为 $2 \pm \sqrt{3}$。
- $x^2 = 5x - 2$ read as $c = -2$ → move first: $x^2 - 5x + 2 = 0$, $c = 2$.
  > $x^2 = 5x - 2$ 读成 $c = -2$ → 先移项：$x^2 - 5x + 2 = 0$，$c = 2$。

The discriminant · 判别式

FORMULAS · 公式

$D = b^2 - 4ac$ — Discriminant of $ax^2 + bx + c = 0$

> $ax^2 + bx + c = 0$ 的判别式

REMEMBER · 记住

- $D = b^2 - 4ac$: $D > 0$ two roots, $D = 0$ one (double) root, $D < 0$ no real roots.
  > $D = b^2 - 4ac$：$D > 0$ 两个根，$D = 0$ 一个（重）根，$D < 0$ 无实数根。
- D a perfect square → rational roots → it factors.
  > D 是完全平方数 → 有理根 → 能因式分解。

WATCH OUT · 当心

- $D = 0$ means "no solution" → it means exactly one real root.
  > $D = 0$ 当成「无解」→ 实际是恰有一个实数根。
- $x^2 + (k + 1)x + 4$: $b = k \to b = k + 1$; square the whole bracket.
  > $x^2 + (k + 1)x + 4$：$b = k$ → 应为 $b = k + 1$，整个括号一起平方。

Choosing a method & word problems · 选方法与应用题

FORMULAS · 公式

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Works for every quadratic

> 对所有二次方程都适用

$a^2 + b^2 = c^2$ — Pythagorean theorem (right-triangle problems)

> 勾股定理（直角三角形问题）

REMEMBER · 记住

- Variable = what is asked; equation from area, Pythagoras or "height = value".
  > 设所求为未知数；由面积、勾股定理或「高度 = 某值」列方程。
- A frame or path of width x adds $2x$ to each dimension.
  > 宽为 x 的边框或小路，使每个方向的长度都增加 $2x$。

WATCH OUT · 当心

- Border of width x: $(10 + x)(6 + x)$ → $(10 + 2x)(6 + 2x)$.
  > 宽为 x 的边框：$(10 + x)(6 + x)$ → 应为 $(10 + 2x)(6 + 2x)$。
- Ball at a height at $t = 1$ and $t = 3$: giving one → give both (up and down).
  > 小球在 $t = 1$ 和 $t = 3$ 都到某高度：只答一个 → 两个都要（上升和下降）。