📈 Graphs & characteristics

图象与特征

Date · 日期 _____ Topic · 知识点 1/5

CUES · QUESTIONS

提示 · 问题

1. Axis of symmetry of $y = ax^2 + bx + c$?

   > $y = ax^2 + bx + c$ 的对称轴?

2. How do you tell max or min from the equation?

   > 怎样从解析式判断有最大值还是最小值?

3. Domain of any quadratic function?

   > 任何二次函数的定义域?

4. Fastest y-intercept of $y = 3x^2 - 5x + 7$?

   > $y = 3x^2 - 5x + 7$ 的 y 截距怎么最快得到?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

NOTES

笔记

1 · Worked example · 1 · 看一道例题 Copy one example from the page. Next to each line write WHY in a few words. · 抄一道例题，每一行旁边用几个字写「为什么」。

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3 · Your turn · 3 · 自己做 A new question. Then check it against the answer. · 做一道新题，然后对答案。

SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

📈 Vertex form & transformations

顶点式与图象变换

Date · 日期 _____ Topic · 知识点 2/5

CUES · QUESTIONS

提示 · 问题

1. Vertex of $y = a(x - p)^2 + q$?
   > $y = a(x - p)^2 + q$ 的顶点?
2. $y = (x + 5)^2$ moves which way?
   > $y = (x + 5)^2$ 往哪边移?
3. Step pattern for $y = x^2$ from the vertex?
   > $y = x^2$ 从顶点出发的步长规律?
4. How many x-intercepts does $y = 3(x - 1)^2 + 2$ have?
   > $y = 3(x - 1)^2 + 2$ 有几个 x 截距?

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

📈 Standard form → vertex form

一般式化顶点式

Date · 日期 _____ Topic · 知识点 3/5

CUES · QUESTIONS

提示 · 问题

1. What do you add to $x^2 + 10x$ to make a perfect square?
   > $x^2 + 10x$ 加多少能配成完全平方?
2. First move for $y = 3x^2 + 12x - 1$?
   > $y = 3x^2 + 12x - 1$ 第一步做什么?
3. Fastest check of your vertex?
   > 最快的顶点验算?
4. Does a change when you convert forms?
   > 换形式时 a 会变吗?

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

📈 Factored form & writing equations

交点式与求解析式

Date · 日期 _____ Topic · 知识点 4/5

CUES · QUESTIONS

提示 · 问题

1. x-intercepts of $y = a(x - r)(x - s)$?
   > $y = a(x - r)(x - s)$ 的 x 截距?
2. Graph touches the x-axis only at $x = 3$. Form?
   > 图象只在 $x = 3$ 处碰到 x 轴，用什么形式?
3. Which form shows the max/min directly?
   > 哪种形式直接显示最值?
4. Zeros -5 and 1. Axis?
   > 零点 -5 和 1，对称轴?

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

📈 Max/min problems

最值应用题

Date · 日期 _____ Topic · 知识点 5/5

CUES · QUESTIONS

提示 · 问题

1. Revenue formula?
   > 收入公式?
2. Max of $y = a(x - r)(x - s)$ happens where?
   > $y = a(x - r)(x - s)$ 在哪里取最值?
3. Two numbers add to 20. Max product?
   > 两数之和为 20，乘积最大是?
4. First move in any optimization word problem?
   > 最值应用题的第一步?

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

📈 Quadratic Functions

二次函数

• Formula and trap sheet

公式与陷阱一页纸

Everything you need before a test, on the fewest pages. Fold it and keep it in your binder. · 考前要看的东西，压缩在尽量少的页数里。折起来放进文件夹。

Graphs & characteristics · 图象与特征

FORMULAS · 公式

$x = -\dfrac{b}{2a}$ — Axis of symmetry of $y = ax^2 + bx + c$

> $y = ax^2 + bx + c$ 的对称轴

$x_{\text{axis}} = \dfrac{r + s}{2}$ — Axis is the midpoint of the x-intercepts r and s

> 对称轴是两个 x 截距 r、s 的中点

REMEMBER · 记住

- $a > 0$ opens up (minimum), $a < 0$ opens down (maximum); the vertex sits on the axis $x = -\frac{b}{2a}$.
  > $a > 0$ 开口向上（有最小值），$a < 0$ 开口向下（有最大值）；顶点在对称轴 $x = -\frac{b}{2a}$ 上。
- Range uses the vertex y-value: $y \geq q$ (up) or $y \leq q$ (down). Domain is always all real numbers.
  > 值域看顶点的纵坐标：开口向上 $y \geq q$，向下 $y \leq q$。定义域永远是全体实数。

WATCH OUT · 当心

- Axis of $y = 2x^2 + 8x$ as $x = 2 \to x = -\frac{8}{2(2)} = -2$ (keep the minus).
  > $y = 2x^2 + 8x$ 的对称轴写成 $x = 2 \to$ 应为 $x = -\frac{8}{2(2)} = -2$（别丢负号）。
- Range from the vertex x-value ($y \geq 2$ for vertex $(2, -5)$) → use the y-value: $y \geq -5$.
  > 用顶点横坐标写值域（顶点 $(2, -5)$ 写成 $y \geq 2$）→ 用纵坐标：$y \geq -5$。

Vertex form & transformations · 顶点式与图象变换

FORMULAS · 公式

$y = a(x - p)^2 + q$ — Vertex form: vertex (p, q), axis $x = p$

> 顶点式：顶点 (p, q)，对称轴 $x = p$

$(x,\ y) \ \to \ (x + p,\ ay + q)$ — Mapping rule from $y = x^2$

> 由 $y = x^2$ 出发的映射规则

REMEMBER · 记住

- $y = a(x - p)^2 + q$: vertex (p, q), axis $x = p$. $(x + 3)^2$ means $p = -3$.
  > $y = a(x - p)^2 + q$：顶点 (p, q)，对称轴 $x = p$。$(x + 3)^2$ 表示 $p = -3$。
- $|a| > 1$ narrower, $|a| < 1$ wider, $a < 0$ flips. From the vertex: over 1, 2, 3 → up a, $4a$, $9a$.
  > $|a| > 1$ 变窄，$|a| < 1$ 变宽，$a < 0$ 翻转。从顶点出发：右移 1、2、3 → 上升 a、$4a$、$9a$。

WATCH OUT · 当心

- Vertex of $y = 2(x + 5)^2 - 1$ as $(5, -1) \to (-5, -1)$.
  > $y = 2(x + 5)^2 - 1$ 的顶点写成 $(5, -1) \to$ 应为 $(-5, -1)$。
- $a = \frac{1}{3}$ called narrower → vertical stretch by $\frac{1}{3}$ makes it wider.
  > $a = \frac{1}{3}$ 说成变窄 → 竖直伸缩因子 $\frac{1}{3}$，图象变宽。

Standard form → vertex form · 一般式化顶点式

FORMULAS · 公式

$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ — The completing-the-square identity

> 配方恒等式

$y = ax^2 + bx + c \ \Rightarrow \ p = -\dfrac{b}{2a},\ q = f(p)$ — Shortcut to the vertex

> 求顶点的捷径

REMEMBER · 记住

- $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
- $a \neq 1$: factor a from the x^2 and x terms only; the subtracted square leaves the bracket times a.
  > $a \neq 1$：只从 x^2 项和 x 项提出 a；减掉的平方数移出括号时要乘 a。

WATCH OUT · 当心

- $3(x^2 - 4x + 4 - 4) + 5 = 3(x - 2)^2 + 1$ → the -4 leaves as -12: $3(x - 2)^2 - 7$.
  > $3(x^2 - 4x + 4 - 4) + 5 = 3(x - 2)^2 + 1 \to -4$ 出括号变成 -12：$3(x - 2)^2 - 7$。
- $-x^2 + 6x = -(x^2 + 6x)$ → $-(x^2 - 6x)$.
  > $-x^2 + 6x = -(x^2 + 6x) \to$ 应为 $-(x^2 - 6x)$。

Factored form & writing equations · 交点式与求解析式

FORMULAS · 公式

$y = a(x - r)(x - s)$ — Factored form: x-intercepts r and s

> 交点式：x 截距为 r 和 s

$x = \dfrac{r + s}{2}$ — Axis of symmetry

> 对称轴

REMEMBER · 记住

- $y = a(x - r)(x - s)$: zeros r, s (opposite signs to the brackets); axis $x = \frac{r+s}{2}$.
  > $y = a(x - r)(x - s)$：零点 r, s（与括号里符号相反）；对称轴 $x = \frac{r+s}{2}$。
- Zeros alone do not fix the parabola: always use a third point to find a.
  > 只有零点定不出抛物线：一定要用第三个点求 a。

WATCH OUT · 当心

- $y = (x + 4)(x - 1)$ zeros $4, -1$ → $-4, 1$.
  > $y = (x + 4)(x - 1)$ 的零点写成 $4, -1$ → 应为 $-4, 1$。
- $3x^2 - 3x - 18 = (x - 3)(x + 2)$ → $3(x - 3)(x + 2)$ (keep a).
  > $3x^2 - 3x - 18 = (x - 3)(x + 2)$ → 应为 $3(x - 3)(x + 2)$（保留 a）。

Max/min problems · 最值应用题

FORMULAS · 公式

$x = -\dfrac{b}{2a}$ — Where the max/min happens

> 取到最值的位置

$R = (\text{price}) \times (\text{quantity})$ — Revenue

> 收入 = 单价 × 销量

REMEMBER · 记住

- Optimize = find the vertex. "When / which value" → x-coordinate; "how much" → y-coordinate.
  > 求最值 = 求顶点。「何时 / 取何值」→ 横坐标；「最多 / 最少是多少」→ 纵坐标。
- Build the function first: area $= lw$, revenue = price $\times$ number sold.
  > 先建函数：面积 = 长 × 宽，收入 = 单价 × 销量。

WATCH OUT · 当心

- "What is the max height?" answered $t = 3$ → give $h(3)$.
  > 「最大高度是多少？」答 $t = 3$ → 应答 $h(3)$。
- Answering $n = 4$ as the price → price is $p_0 + 4$.
  > 把 $n = 4$ 当成价格 → 价格是 $p_0 + 4$。