# Greatest common factor & grouping

## 提公因式与分组分解

Date · 日期 _____ Topic · 知识点 1/4

### CUES · QUESTIONS

提示 · 问题

1. First step in ANY factoring question?
   > 任何分解题的第一步?
2. GCF of $12a^3b^2$ and $18a^2b^5$?
   > $12a^3b^2$ 与 $18a^2b^5$ 的公因式?
3. Four terms, no GCF. Try...
   > 四项且没有公因式，试……
4. $(5 - x)$ written with $x$ first?
   > $(5 - x)$ 让 $x$ 在前怎么写?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

### NOTES

笔记

**1 · Worked example · 1 · 看一道例题** Copy one example from the page. Next to each line write WHY in a few words. · 抄一道例题，每一行旁边用几个字写「为什么」。

**2 · Fill in the blanks · 2 · 填空题** Cover the steps and redo it. Leave a gap where you are unsure. · 盖住步骤自己重做，不确定的地方先空着。

**3 · Your turn · 3 · 自己做** A new question. Then check it against the answer. · 做一道新题，然后对答案。

### SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

# Factoring trinomials $ax^2+bx+c$

## 三项式 $ax^2+bx+c$ 的分解

Date · 日期 _____ Topic · 知识点 2/4

### CUES · QUESTIONS

提示 · 问题

1. Decomposition: which two numbers?
   > 拆项法找哪两个数?
2. $c > 0$, $b < 0$: signs of the two numbers?
   > $c > 0$，$b < 0$：两个数的符号?
3. Factor $x^2 + 9x + 20$.
   > 分解 $x^2 + 9x + 20$。
4. Leading coefficient negative — first move?
   > 首项系数为负——第一步?

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### SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

# Difference of squares & perfect squares

平方差与完全平方

Date · 日期 _____ Topic · 知识点 3/4

CUES · QUESTIONS

提示 · 问题

1. $A^2 - B^2$ = ?

   > $A^2 - B^2$ =?

2. Test for a perfect square trinomial?

   > 怎样判断完全平方式?

3. Factor $4x^2 - 1$.

   > 分解 $4x^2 - 1$。

4. Does $x^2 + 16$ factor?

   > $x^2 + 16$ 能分解吗?

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NOTES

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

# 🧩 Factoring by substitution

## 换元法分解

Date · 日期 _____ Topic · 知识点 4/4

### CUES · QUESTIONS

提示 · 问题

1. Substitution for $x^4 + 3x^2 - 10$?
   > $x^4 + 3x^2 - 10$ 怎么换元?
2. Spot quadratic form how?
   > 怎样认出二次型?
3. Last step after factoring in $m$?
   > 关于 $m$ 分解后最后一步?
4. $(x+5)^2 - 16$ = ?
   > $(x+5)^2 - 16 =$ ?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

### NOTES

笔记

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### SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

# 🧩 Polynomial Factoring

多项式因式分解

- **Formula and trap sheet**

公式与陷阱一页纸

Everything you need before a test, on the fewest pages. Fold it and keep it in your binder. · 考前要看的东西，压缩在尽量少的页数里。折起来放进文件夹。

## Greatest common factor & grouping · 提公因式与分组分解

### FORMULAS · 公式

$ab + ac = a(b + c)$ — Common factor (distributive law backwards)

> 提公因式（分配律倒过来用）

$ax + ay + bx + by = a(x + y) + b(x + y) = (x + y)(a + b)$ — Factoring by grouping

> 分组分解

### REMEMBER · 记住

- GCF first: largest number dividing every coefficient × each variable to its LOWEST power.
  > 先提公因式：能整除所有系数的最大数 × 每个字母的最低次幂。
- A bracket can be the common factor: $2x(x - 3) + 5(x - 3) = (x - 3)(2x + 5)$.
  > 整个括号也能当公因式：$2x(x - 3) + 5(x - 3) = (x - 3)(2x + 5)$。

### WATCH OUT · 当心

- A term equal to the GCF leaves 1: $3x^2y + 6x^3y = 3x^2y(1 + 2x)$, not $3x^2y(2x)$.
  > 和公因式相同的项除完剩 1：$3x^2y + 6x^3y = 3x^2y(1 + 2x)$，不是 $3x^2y(2x)$。
- $-9x - 18 = -9(x - 2)$ × → $-9(x + 2)$.
  > $-9x - 18 = -9(x - 2)$ × → $-9(x + 2)$。

## Factoring trinomials ax²+bx+c · 三项式 ax²+bx+c 的分解

### FORMULAS · 公式

$x^2 + (m + n)x + mn = (x + m)(x + n)$ — Trinomial with $a = 1$

> 首项系数为 1 的三项式

$ax^2 + bx + c$ : $mn = ac$, $m + n = b \Rightarrow ax^2 + mx + nx + c$ — Decomposition: split the middle term

> 拆项法：把中间项拆开

### REMEMBER · 记住

- $a = 1$: two numbers that multiply to $c$ and add to $b$.
  > $a = 1$：找两个数，积为 $c$、和为 $b$。
- $a \neq 1$: two numbers that multiply to $ac$ and add to $b$; split the middle, then group.
  > $a \neq 1$：找积为 $ac$、和为 $b$ 的两个数，拆中间项再分组。

### WATCH OUT · 当心

- $6x^2 + 7x - 3$: product $-3$ × → product $ac = -18$.
  > $6x^2 + 7x - 3$：积用 $-3$ × → 积应是 $ac = -18$。
- $2x^2 + 10x + 12 = (2x + 4)(x + 3)$ is not fully factored → $2(x + 2)(x + 3)$.
  > $2x^2 + 10x + 12 = (2x + 4)(x + 3)$ 没分解完 → $2(x + 2)(x + 3)$。

## Difference of squares & perfect squares · 平方差与完全平方

### FORMULAS · 公式

$A^2 - B^2 = (A - B)(A + B)$ — Difference of squares

> 平方差公式

$A^2 \pm 2AB + B^2 = (A \pm B)^2$ — Perfect square trinomials

> 完全平方公式

### REMEMBER · 记住

- $A^2 - B^2 = (A - B)(A + B)$: two squares with a MINUS.
  > $A^2 - B^2 = (A - B)(A + B)$：两个平方数，中间是减号。
- $A^2 \pm 2AB + B^2 = (A \pm B)^2$: check the middle is exactly $2AB$.
  > $A^2 \pm 2AB + B^2 = (A \pm B)^2$：一定要验证中间项正好是 $2AB$。

### WATCH OUT · 当心

- $x^2 + 9 = (x + 3)^2$ × → a sum of squares does not factor.
  > $x^2 + 9 = (x + 3)^2$ × → 平方和不能分解。
- $x^2 + 10x + 16 = (x + 4)^2$ × → middle would be $8x$; it is $(x + 2)(x + 8)$.
  > $x^2 + 10x + 16 = (x + 4)^2$ × → 那样中间项是 $8x$；应为 $(x + 2)(x + 8)$。

## Factoring by substitution · 换元法分解

### FORMULAS · 公式

$a\,[f(x)]^2 + b\,f(x) + c \xrightarrow{m = f(x)} am^2 + bm + c$ — Substitution for quadratic form

> 二次型的换元

### REMEMBER · 记住

- Spot $a(\text{stuff})^2 + b(\text{stuff}) + c$; let $m =$ the stuff.
  > 认出 $a(\text{整体})^2 + b(\text{整体}) + c$；令 $m =$ 这个整体。
- Factor in $m$, substitute back, simplify each bracket.
  > 按 $m$ 分解，代回，再化简每个括号。

### WATCH OUT · 当心

- Leaving the answer in $m$ × → substitute back to $x$.
  > 答案停在 $m$ × → 要代回 $x$。
- $(2x - 1) - (x + 3) = x + 2$ × → $x - 4$: the minus hits the whole bracket.
  > $(2x - 1) - (x + 3) = x + 2$ × → $x - 4$：减号作用于整个括号。