√ Simplifying & ordering radicals

根式化简与比较大小

Date · 日期 _____ Topic · 知识点 1/3

CUES · QUESTIONS

提示 · 问题

1. Simplify $\sqrt{50}$.
   化简 $\sqrt{50}$。
2. Write $2\sqrt[3]{5}$ as an entire radical.
   把 $2\sqrt[3]{5}$ 写成整根式。
3. Perfect squares to know by heart?
   要背熟的完全平方数?
4. First move to order $5\sqrt{2}$ and $4\sqrt{3}$?
   比较 $5\sqrt{2}$ 与 $4\sqrt{3}$ 的第一步?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

NOTES

笔记

1 · Worked example · 1 · 看一道例题 Copy one example from the page. Next to each line write WHY in a few words. · 抄一道例题，每一行旁边用几个字写「为什么」。

2 · Fill in the blanks · 2 · 填空题 Cover the steps and redo it. Leave a gap where you are unsure. · 盖住步骤自己重做，不确定的地方先空着。

3 · Your turn · 3 · 自己做 A new question. Then check it against the answer. · 做一道新题，然后对答案。

SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

√ Operations with radicals

根式的四则运算

Date · 日期 _____ Topic · 知识点 2/3

CUES · QUESTIONS

提示 · 问题

1. Conjugate of $5-\sqrt{7}$?
   > $5-\sqrt{7}$ 的共轭式?
2. $\sqrt{a}\cdot\sqrt{a}$ = ?
   > $\sqrt{a}\cdot\sqrt{a}$=?
3. When can you add two radicals?
   > 什么时候两个根式能相加?
4. First move on $\dfrac{k}{a+\sqrt{b}}$?
   > 见到 $\dfrac{k}{a+\sqrt{b}}$ 第一步?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

√ Radical equations & extraneous roots

根式方程与增根

Date · 日期 _____ Topic · 知识点 3/3

CUES · QUESTIONS

提示 · 问题

1. What is an extraneous root?
   > 什么是增根?
2. First move on $\sqrt{2x+1}-3=x$?
   > 见到 $\sqrt{2x+1}-3=x$ 第一步?
3. Restriction for $\sqrt{6-2x}$?
   > $\sqrt{6-2x}$ 的限制条件?
4. Do cube-root equations make extraneous roots?
   > 立方根方程会产生增根吗?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

NOTES

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SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

√ Radicals

根式

• Formula and trap sheet

公式与陷阱一页纸

Everything you need before a test, on the fewest pages. Fold it and keep it in your binder. · 考前要看的东西，压缩在尽量少的页数里。折起来放进文件夹。

Simplifying & ordering radicals · 根式化简与比较大小

FORMULAS · 公式

$\sqrt{ab} = \sqrt{a}\,\sqrt{b} \quad (a, b \geq 0)$ — Product rule for radicals

> 根式乘法法则

$k\sqrt[n]{a} = \sqrt[n]{k^n a}$ — Mixed ↔ entire radical

> 最简根式与整根式互化

REMEMBER · 记住

- Pull out the LARGEST perfect square (perfect cube for $\sqrt[3]{}$): $\sqrt{72} = 6\sqrt{2}$.
  > 提出最大的完全平方因数（立方根提完全立方数）：$\sqrt{72} = 6\sqrt{2}$。
- Mixed → entire: square the outside number and multiply it in: $3\sqrt{5} = \sqrt{45}$.
  > 最简根式化为整根式：把外面的数平方后乘进去：$3\sqrt{5} = \sqrt{45}$。

WATCH OUT · 当心

- $\sqrt{72} = 2\sqrt{18}$ is not finished → $6\sqrt{2}$.
  > $\sqrt{72} = 2\sqrt{18}$ 没化完 → $6\sqrt{2}$。
- $\sqrt{9+16} = 3+4$ ✗ → $\sqrt{25} = 5$. Roots split over $\times$, never over $+$.
  > $\sqrt{9+16} = 3+4$ ✗ → $\sqrt{25} = 5$。根号只能对乘法拆，不能对加法拆。

Operations with radicals · 根式的四则运算

FORMULAS · 公式

$m\sqrt[n]{a} \cdot k\sqrt[n]{b} = mk\sqrt[n]{ab}$ — Multiplying radicals (same index)

> 同次根式相乘

$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = a - b$ — Conjugates multiply to a rational number

> 共轭式相乘得有理数

REMEMBER · 记住

- Add/subtract LIKE radicals only — simplify each first: $\sqrt{12} + \sqrt{27} = 5\sqrt{3}$.
  > 只有同类根式才能加减——先逐个化简：$\sqrt{12} + \sqrt{27} = 5\sqrt{3}$。
- Multiply outside × outside, inside × inside; expand brackets like FOIL.
  > 相乘：系数乘系数，根号里乘根号里；括号像多项式一样展开。

WATCH OUT · 当心

- $\sqrt{2} + \sqrt{3} = \sqrt{5}$ ✗ → unlike radicals stay as they are.
  > $\sqrt{2} + \sqrt{3} = \sqrt{5}$ ✗ → 不同类根式不能合并。
- $(3-\sqrt{2})^2 = 9 + 2$ ✗ → $9 - 6\sqrt{2} + 2 = 11 - 6\sqrt{2}$.
  > $(3-\sqrt{2})^2 = 9+2$ ✗ → $9 - 6\sqrt{2} + 2 = 11 - 6\sqrt{2}$。

Radical equations & extraneous roots · 根式方程与增根

FORMULAS · 公式

$\sqrt{A} = B \;\Rightarrow\; A = B^2, \quad \text{valid only if } B \geq 0$ — Squaring step and the hidden condition

> 平方这一步及隐含条件

REMEMBER · 记住

- Restrictions → ISOLATE the radical → raise to the index → solve → CHECK in the original.
  > 写限制条件 → 孤立根式 → 两边乘方 → 解方程 → 代回原方程检验。
- Squaring can create extraneous roots: reject any root that fails the original.
  > 平方可能产生增根：不满足原方程的根要舍去。

WATCH OUT · 当心

- Squaring before isolating: $(\sqrt{x}+1)^2 = x+1$ ✗ → $x + 2\sqrt{x} + 1$.
  > 没孤立就平方：$(\sqrt{x}+1)^2 = x+1$ ✗ → $x + 2\sqrt{x} + 1$。
- $(2\sqrt{x+4})^2 = 2(x+4)$ ✗ → $4(x+4)$.
  > $(2\sqrt{x+4})^2 = 2(x+4)$ ✗ → $4(x+4)$。