🔢 The real number system

实数系统

Date · 日期 _____ Topic · 知识点 1/3

CUES · QUESTIONS

提示 · 问题

1. Order the sets from smallest to largest.
   > 数集从小到大怎么排?
2. Decimal test for rational?
   > 用小数判断有理数?
3. Is $\sqrt{-16}$ a real number?
   > $\sqrt{-16}$ 是实数吗?
4. Write $0.\overline{7}$ as a fraction.
   > 把 $0.\overline{7}$ 写成分数。

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

NOTES

笔记

1 · Worked example · 1 · 看一道例题 Copy one example from the page. Next to each line write WHY in a few words. · 抄一道例题，每一行旁边用几个字写「为什么」。

2 · Fill in the blanks · 2 · 填空题 Cover the steps and redo it. Leave a gap where you are unsure. · 盖住步骤自己重做，不确定的地方先空着。

3 · Your turn · 3 · 自己做 A new question. Then check it against the answer. · 做一道新题，然后对答案。

SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

🔢 Exponent laws (zero & negative)

指数律（零指数与负指数）

Date · 日期 _____ Topic · 知识点 2/3

CUES · QUESTIONS

提示 · 问题

1. $a^m \cdot a^n = ?$ $(a^m)^n = ?$

   $a^m \cdot a^n = ?$ $(a^m)^n = ?$

2. a^{-n} means…

   a^{-n} 表示……

3. $\left(\frac{3}{5}\right)^{-2}$ = ?

   $\left(\frac{3}{5}\right)^{-2} = ?$

4. $(7x)^0$ vs $7x^0$?

   $(7x)^0$ 和 $7x^0$?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

NOTES

笔记

1 · Worked example · 1 · 看一道例题 Copy one example from the page. Next to each line write WHY in a few words. · 抄一道例题，每一行旁边用几个字写「为什么」。

2 · Fill in the blanks · 2 · 填空题 Cover the steps and redo it. Leave a gap where you are unsure. · 盖住步骤自己重做，不确定的地方先空着。

3 · Your turn · 3 · 自己做 A new question. Then check it against the answer. · 做一道新题，然后对答案。

SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

🔢 Rational exponents

分数指数

Date · 日期 _____ Topic · 知识点 3/3

CUES · QUESTIONS

提示 · 问题

1. $a^{m/n}$ as a radical?
   > $a^{m/n}$ 写成根式?
2. $32^{3/5}$ = ?
   > $32^{3/5}$ =?
3. Root first or power first?
   > 先开方还是先乘方?
4. $\sqrt[4]{x} \cdot \sqrt{x}$ as one power?
   > $\sqrt[4]{x} \cdot \sqrt{x}$ 写成一个幂?

Cover the notes. Answer each cue from memory. · 盖住右边的笔记，凭记忆回答每个提示。

NOTES

笔记

1 · Worked example · 1 · 看一道例题 Copy one example from the page. Next to each line write WHY in a few words. · 抄一道例题，每一行旁边用几个字写「为什么」。

2 · Fill in the blanks · 2 · 填空题 Cover the steps and redo it. Leave a gap where you are unsure. · 盖住步骤自己重做，不确定的地方先空着。

3 · Your turn · 3 · 自己做 A new question. Then check it against the answer. · 做一道新题，然后对答案。

SUMMARY — IN MY OWN WORDS, TWO SENTENCES

小结——用自己的话，两句

🔢 Real Numbers & Powers

实数与幂

- **Formula and trap sheet**

公式与陷阱一页纸

Everything you need before a test, on the fewest pages. Fold it and keep it in your binder. · 考前要看的东西，压缩在尽量少的页数里。折起来放进文件夹。

The real number system · 实数系统

FORMULAS · 公式

$\mathbb{N} \subset \mathbb{W} \subset \mathbb{I} \subset \mathbb{Q} \subset \mathbb{R}, \qquad \mathbb{R} = \mathbb{Q} \cup \overline{\mathbb{Q}}$ — How the number families fit together

> 数集之间的包含关系

$0.\overline{ab} = \dfrac{ab}{99}$ — A two-digit repeating block over 99

> 两位循环节：循环节除以 99

REMEMBER · 记住

- $\mathbb{N} \subset \mathbb{W} \subset \mathbb{I} \subset \mathbb{Q} \subset \mathbb{R}$; 0 is whole but not natural.
  > $\mathbb{N} \subset \mathbb{W} \subset \mathbb{I} \subset \mathbb{Q} \subset \mathbb{R}$；$0$ 是非负整数（$\mathbb{W}$），但不是自然数。
- Rational = $\frac{a}{b}$ with integers a, b: decimal ends or repeats. Irrational: never ends, never repeats.
  > 有理数＝两个整数之比 $\frac{a}{b}$，小数会终止或循环；无理数的小数不终止也不循环。

WATCH OUT · 当心

- $\frac{\pi}{2}$ "looks like a fraction" → rational ×. Top must be an integer: it is irrational.
  > $\frac{\pi}{2}$「长得像分数」所以是有理数 ×。分子必须是整数，它是无理数。
- Root sign → irrational ×. $\sqrt{16} = 4$ is an integer.
  > 有根号就是无理数 ×。$\sqrt{16} = 4$ 是整数。

Exponent laws (zero & negative) · 指数律（零指数与负指数）

FORMULAS · 公式

$a^m \cdot a^n = a^{m+n}, \quad \dfrac{a^m}{a^n} = a^{m-n}, \quad (a^m)^n = a^{mn}$ — Product, quotient, power laws

> 同底数幂的乘法、除法、幂的乘方

$(ab)^n = a^n b^n, \quad \left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ — Power of a product / quotient

> 积的乘方、商的乘方

$a^0 = 1, \quad a^{-n} = \dfrac{1}{a^n}, \quad \left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$ — Zero and negative exponents $(a, b \neq 0)$

> 零指数与负指数（$a, b \neq 0$）

REMEMBER · 记住

- Same base: multiply → add exponents, divide → subtract, power of a power → multiply.
  > 同底数：相乘指数相加，相除指数相减，乘方的乘方指数相乘。
- An outside power hits every factor, numbers too: $(3x^2)^3 = 27x^6$.
  > 括号外的指数作用到每个因式上，系数也要乘方：$(3x^2)^3 = 27x^6$。

WATCH OUT · 当心

- $2^{-3} = -8$ × → $2^{-3} = \frac{1}{8}$.
  > $2^{-3} = -8$ × → $2^{-3} = \frac{1}{8}$。
- $-3^2 = 9$ × → $-3^2 = -9$, but $(-3)^2 = 9$.
  > $-3^2 = 9$ × → $-3^2 = -9$，而 $(-3)^2 = 9$。

Rational exponents · 分数指数

FORMULAS · 公式

$a^{\frac{1}{n}} = \sqrt[n]{a}, \qquad a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$ — Rational exponent ↔ radical

> 分数指数与根式互化

$a^{-\frac{m}{n}} = \dfrac{1}{a^{\frac{m}{n}}}$ — Negative rational exponent $(a \neq 0)$

> 负分数指数（$a \neq 0$）

REMEMBER · 记住

- $a^{m/n} = (\sqrt[n]{a})^m$: bottom of the exponent = root, top = power.
  > $a^{m/n} = (\sqrt[n]{a})^m$：指数的分母是开方次数，分子是乘方次数。
- By hand, take the root first: $27^{2/3} = 3^2 = 9$.
  > 手算先开方，数字小：$27^{2/3} = 3^2 = 9$。

WATCH OUT · 当心

- $8^{2/3}$ = square root cubed × → cube root squared: $2^2 = 4$.
  > $8^{2/3}$＝开平方再立方 × → 开立方再平方：$2^2 = 4$。
- $(-16)^{1/2} = -4$ × → not real; but $(-8)^{1/3} = -2$ is fine (odd root).
  > $(-16)^{1/2} = -4$ × → 不是实数；而 $(-8)^{1/3} = -2$ 没问题（奇次方根）。